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Extending Decentralized Kalman Filtering Results for Novel Real-Time Multisensor Image Fusion and/or Restoration

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Abstract

We pursue the idea that recent "decentralized" Kalman filter (KF) technology, by outfitting each participating imaging sensor with its own dedicated 2-D Kalman filter can be used as the basis of a sensor fusion methodology that allows a final collating filter to assemble the data from diverse imaging sensors of various resolutions into a single resulting image that combines all the available information (in analogy to what is already routinely done in multisensor NAVIGATION applications). The novelty is in working out the theoretical details for 2-D filtering situations while assuming that the image registration problem has already been independently handled beforehand. We synchronize frame size and location of pixels of interest to be comparably located with same "raster scan" speed and size used for each to match up for different sensors. Rule for linear Kalman filters with only Gaussian noises is that the combining of underlying measurements or sensor information can only help and never hurt. We interpret this approach as using several common views of the same scene, as instantaneously obtained from different sensors, all being stacked up vertically one on top of the other, each with its own local 2-D Kalman-like image restoration filter proceeding to raster scan (in multi-layer sync). Then apply the multi-filter combining rules from Decentralized filtering to the bunch to obtain a single best estimate image as the resulting output as a convenient methodology to achieve sensor fusion.

Keywords: Decentralized Kalman Filters, Kalman Filters for 2-D random Fields, Data Fusion Combining/Collating Rules, Extended Kalman filter for parameter identification, ARMA-to-AR Conversion, Adaptive processing.

1 INTRODUCTION

In the past [27], we have reviewed the down-looking GaAs laser line-scanner capabilities of the near-infrared/visible wavelength (NIRV) active/passive subsystem, a component of the entire multispectral active/passive line scanner (MAPLS), which were perceived to be likely candidates to be used for image orientation purposes for data collection. Both the forward-looking CO_2 laser and the $GaAs$ laser line-scanner currently have real-time displays [1]. The $GaAs$ laser line-scanner has a resolution of 1 millirad × 1 millirad at a nominal aircraft surveillance altitude of from 500 to 750 ft (corresponding explicitly to 15 cm and 21 cm resolutions, respectively) for the special mission of this particular Grumman Gulfstream aircraft. We also compared this data collection situation in [27] with that of more standard surveillance aircraft usually operating at much higher altitudes.

From a well-known prior precedent by Les Novak [2], a 35 MHz microwave Synthetic Aperture Radar (SAR), used for post-process imaging with 1 ft × 1 ft resolutions, uses arrays of dihedral (double bounce) and trihedral (triple bounce) corner cubes (of 3 different sizes for 3 orders of magnitude dB_{sm} polarimetric calibrations) that civil engineers had to survey, cross-align, and orient accurately en masse (in maintaining the original polarizations within the reflections) to simultaneously exhibit their "sweet spot" registration flashes in order to stick up above the considerable ground clutter floor to enable successful removal of anomalous speckle. Less clutter/speckle is expected to be present in the above laser radar bands [3].

Typically, Forward Looking InfraRed (FLIR) sensors generate averaged outputs of an array of infrared detectors as they are mechanically scanned through a limited field-of-view (FOV). The most fundamental output at an instant

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of time corresponds to an averaged intensity, as measured over one picture element (pixel). The horizontal and vertical scanning of the detectors through the FLIR FOV results in an array of pixels called a frame of data (at typical frame-completion rates of about 30 Hz). Because of this rapid frame rate in providing IR measurements, attention can frequently be confined for tactical target-tracker applications to a sub-array of pixels (encompassing the target) considerably smaller than the total frame. By utilizing this restricted FOV, the data processing burden can be maintained within the practical real-time throughput and memory constraints of available computational resources, while at the same time focusing only on that subset of the IR information that is actually relevant to the tracking of the particular target. (An 8×8 array consisting of 64 pixels is a typical size for target tracking windows.)

Standard correlation-type algorithms first store a complete set of intensity data, as measured by an IR sensor array at a particular time instant. This set is then cross-correlated with frames generated subsequently. However, otherwise rigorously substantiated and theoretically justified noise averaging (for SNR enhancement of the target signature) is hindered somewhat in a few airborne applications of pure correlation-type-algorithms because the motion of the sensor platform is not sufficiently compensated for by an on-board Inertial Navigation System (INS) or other navaid. The result is a smeared background that is no longer stationary with corrupting noise that is probably also nonstationary. Thus, there is considerable impetus to dispense with conventional frequency-domain correlation-type algorithms, valid only for statistically stationary situations, and to now embrace time-domain algorithms [1], as offered here, in order to reasonably handle these challenging situations without resorting to the inordinate computational burden of bispectrum and trispectrum (i.e., higher-order spectra corresponding in the transform domain to higher-order moments or cumulant) techniques, which can handle nonstationarity but which also demand that the corrupting noises be non-Gaussian (or else they won't work).

Image restoration, as addressed here (in contradistinction to image enhancement which just uses signal processing rules to clean up images), utilizes a mathematical model of the mechanisms that introduce the distortion/degradation and seeks to apply an inverse procedure to recapture/reconstruct the original scene. The two main approaches that can be used to solve this problem are either autoregressive moving average ARMA-based or Kalman-like filter-based (with preference given here to the latter but with further discussion also provided in Sec. 2.3 to dispel a common pervasive historical misconception that has crept into the standard use of ARMA-to-AR conversions, namely, that a finite order ARMA only converts to an infinite order AR [we provide an explicit counterexample to the previous claim]). In particular, the main goal is to compensate for those distortions due to <u>blur</u> and <u>noise</u>.

<u>Blur</u> arises due to:

- Uncompensated relative motion between object and imaging sensor;
- Sensor not being in the focal plane of the lens (i.e., image is out of focus);
- Possible presence of atmospheric turbulence causing distortion in aerial images;
- Presence of imaging system aberrations (imperfections in mating components of lens, sensor, digitizer).

<u>Noise</u> arises from several sources such as: (i) Electronic [thermal motion of electrons in components: sensors, receivers, amplifiers, ...]; (ii) Photoelectric [quantum statistical nature of light and the underlying conversion process in the sensor/transducer]; (iii) Quantization noise [incurred during the digitization process]; (iv) [Possible] film grain noise [randomness in placement location of silver halide grains that record the image]. Other difficulties to be overcome: Sensors and scanners have known nonlinear characteristics that can be represented as point nonlinearities of known form to be compensated for. Our limited goals here in using Kalman filter-based technology is merely (1) to correct for residual uncompensated relative motion between target and sensor focal plane [4] [2] and (2) to compensate for any imaging sensor mis-focusing present (that can be modeled) and (3) to give the best results possible in the acknowledged presence of the above enumerated noises.

Usually assumptions must be made on the nature or cause of any blur observed, the nature of dominant underlying noises significantly affecting the image (qualitative information), and the presumed statistics (quantitative information) of the ideal image (without these corrupting effects present) and some pre-determined error criterion (usually Mean Square Error) to be minimized as the goal (which may not agree with a human's perception of an improved

[1] Kalman filters are also known for their ability to properly handle *nonstationary* white Gaussian noise (WGN) as long as the second-order statistics are either known a priori or are estimated on-line over a sliding data window (yielding only coarse approximations to actual time-varying statistics).

[2] Alternate approaches and other applications exist, such as that discussed in [33].

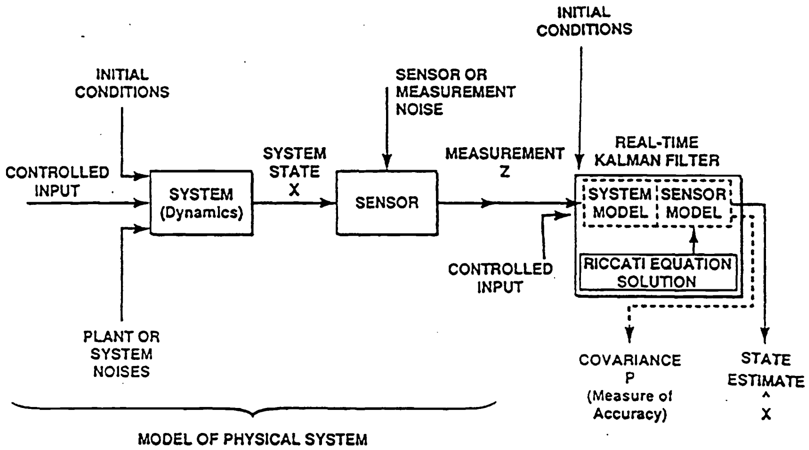

Figure 1: High-Level View of the Internal Structure of a Kalman Filter

representation of the image). A problem is that parameters of blur and image models are usually NOT KNOWN a priori, so it is important to try to identify these parameters (using on-line identification algorithms) directly from the observed image (as a first stage of a two stage [perhaps iterative/recursive process] that can eventually be described in toto as an adaptive filtering procedure, where the second stage that follows is the more familiar Kalman filter-like portion [39]–[49], but now to be generalized and implemented for 2-D). The 2-D version of Kalman filtering typically uses a "raster scan" or "strip" updating rule (sometimes computationally simplified so that only elements within some relatively small neighborhood of the current pixel of interest are updated *under the assumption* that pixels outside the update region are insignificantly correlated with it). The good news is, according to [17, p.18, col. 1 following Eq. 8], that simplifications accrue in the case of uniform motion blur and out-of-focus blur and the effect of both of these degradations can be represented by only one parameter, namely: the *extent* of the motion blur and the *radius* of the disc that represents the sensor point spread function (PSF), respectively. Other more academically esoteric aspects of distributed sensor integration are treated in [5].

2 STATUS OF EVOLVING KALMAN FILTER TECHNOLOGY (the main working tool to obtain NEW image fusion results)

Several events of note regarding the above stated topic have occurred within the last eight years. These events will be ordered here to provide a simplified overview as a timely assessment of the status of events. These events are perceived to be potentially significant in image restoration, as discussed herein, and further can serve as a basis for multisensor fusion (just as the centralized Kalman filter has already successfully served in this role for three decades as the basis for real-time Navigation information fusion from diverse navaid sensors of differing accuracies and sample rates [35]-[37, p. 274] and has already been recast to reap the considerable benefits of a decentralized Kalman filter architecture [6]-[12]). See Figs. 1, 2, 3 and 4 for a self-explanatory visual perspective and motivation for why this topic is so important.

The gist of other related topics is explained here that interface with above sensor fusion area, where significant events have also occurred to offer new results that will likely be exploited in this pursuit. Except for the lead discussion in Sec. 2.1 appearing next, most of the gory mathematical derivation from first principles has been dispensed with here in favor of instead conveying only the underlying ideas and physical motivations (otherwise it wouldn't fit within reasonable page constraints).

The topic of the next section derives its importance from the direct impact it has on image restoration and its potential in sensor fusion applications via the novel approach offered in Sec. 4 below, which depends explicitly on the status of Decentralized Filtering as a necessary precursor.

	PROPAGATE STEP	UPDATE STEP
COVARIANCE	$P_{k\|k-1} = \Phi(k, k-1)P_{k-1\|k-1}\Phi^T(k, k-1) + Q_k$	$P_{k\|k} = [I - K_k H_k]P_{k\|k-1}[I - K_k H_k]^T + K_k R K_k^T$
FILTER GAIN	$K_k = P_{k\|k-1}H_k^T[H_k P_{k\|k-1} H_k^T + R]^{-1}$	
FILTER	$\hat{x}_{k\|k-1} = \hat{x}_{k-1\|k-1} + \int_{t_{k-1}}^{t_k} f(\hat{x}_\tau \| t_{k-1})\, d\tau$	$\hat{x}_{k\|k} = \hat{x}_{k\|k-1} + K_k(z_k - h(\hat{x}_{k\|k-1}))$

Table 1: Extended Kalman Filter (EKF) Implementation/Mechanization Equations, where gradients of nonlinear ODE are "linearized" on-line about the estimate

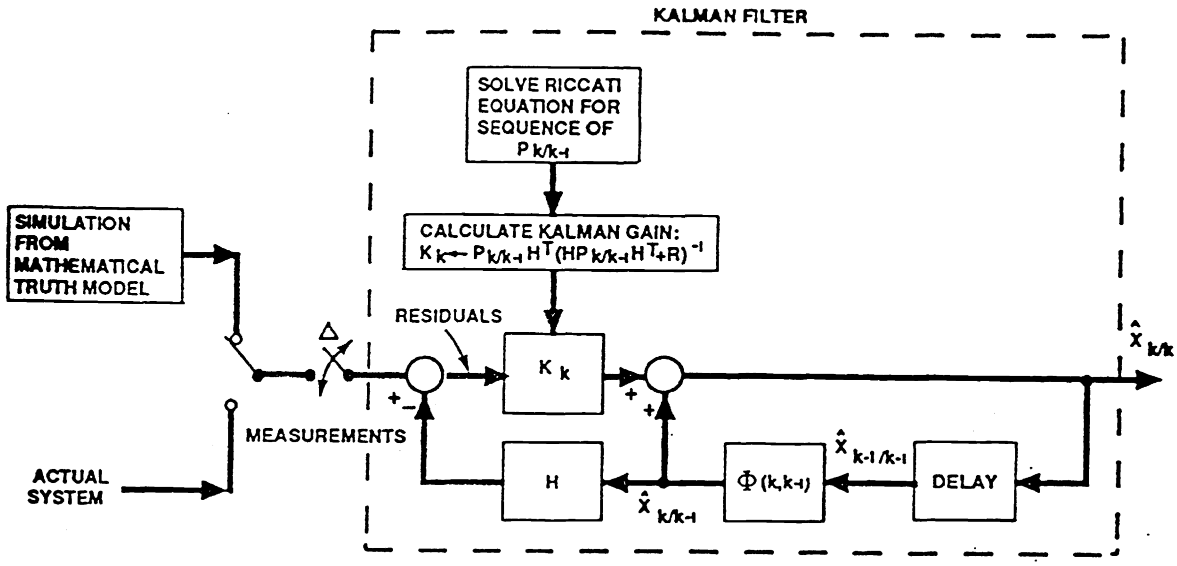

Figure 2: A Standard 1-D Discrete-Time Kalman Filter Mechanization

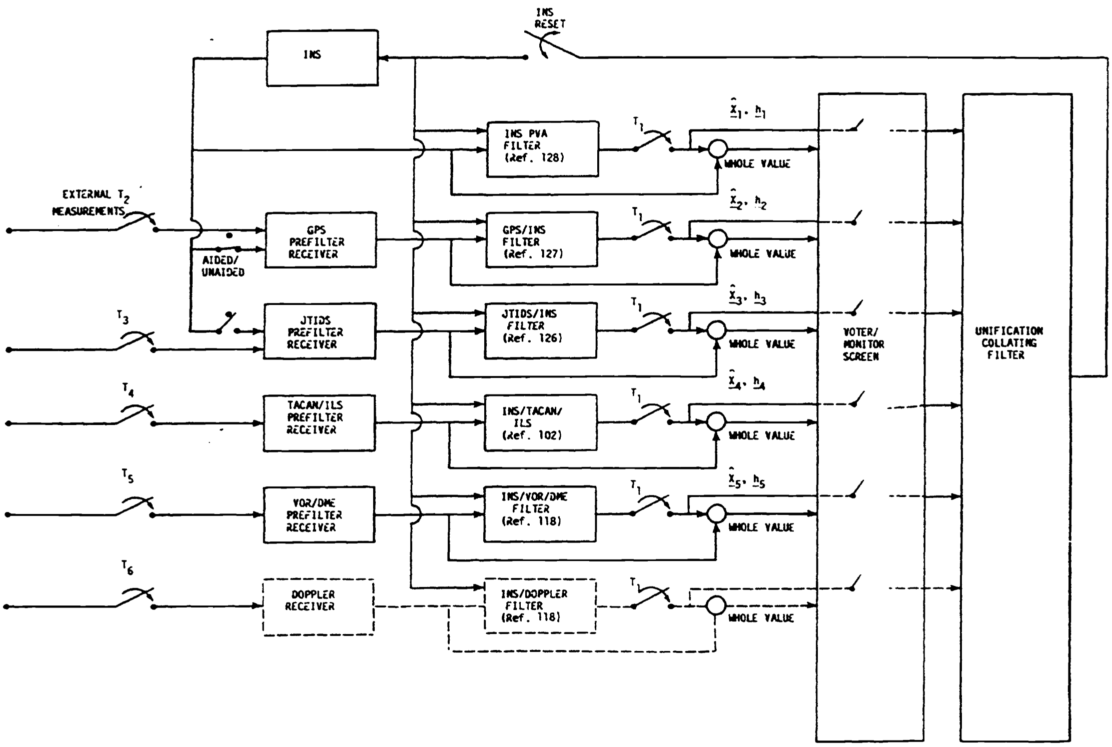

Figure 3: Decentralized Semi-autonomous Multisensor Navigation (SMN) filter to enhance failure detection/isolation and to ease reconfiguration

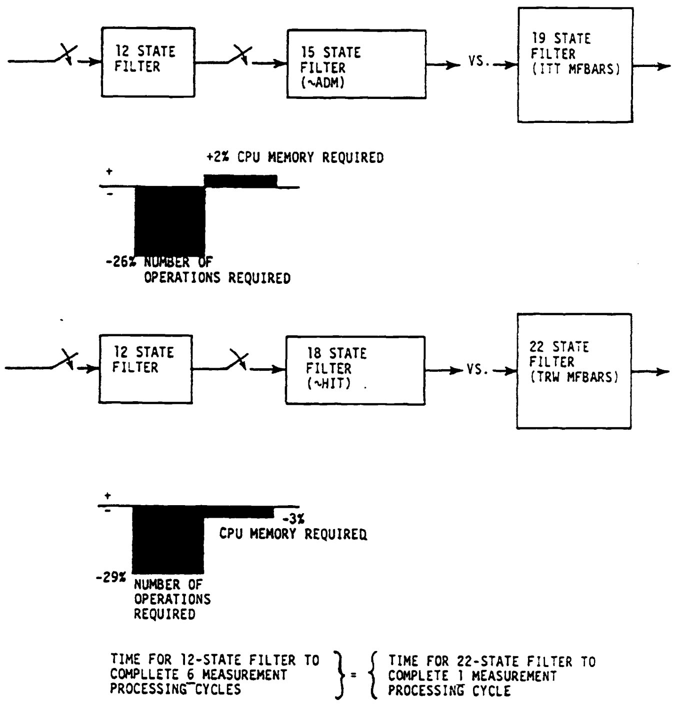

Figure 4: Benefits of two filters over one federated filter (for GPS/JTIDS/INS example)

2.1 A common thread occurring within three supposedly different 1-D decentralized Kalman filter mechanizations

Three independent teams of investigators use the same *alternate form* [3] of the "centralized" KF implementation equations that they select as a jumping off point for generalization [4] to the following diverse applications:

- Multi-sensor camera data fusion for robotics and/or telerobots [54];
- Target tracking using data from non-geographically co-located sensors with coupling via noisy communication lines [55] [5](cf., [53]);
- Multi-sensor *integrated* navigation [57] [6].

All of the above three investigations now use the alternate KF form (depicted here in Table 2) for computing the Kalman gain K_k (in [54, Eq. 7], [55, Eq. 7b], [57, Eq. 45]) involving use of the covariance update $P_{k|k}$ instead of using the usual predicted covariance $P_{k|k-1}$. While these references do display their final mechanization equations, they don't show its derivation, which can be elusive and nonobvious so we offer our derivation below to expose important details.

The alternate form for centralized KF mechanization (serving as the fundamental stepping stone or jumping off point in [54]-[57] for eventual generalization to the decentralized filtering case) has a theoretical twist that is utilized within this structure, as reviewed next. The measurement records collected by the multiple 1-D decentralized sensors $(i = 1, \ldots, N)$ can be summarized in aggregate block form as measurements:

$$z(k) = \left[z_1^T(k), \ldots, z_N^T(k)\right]^T \ , \tag{1}$$

and as an effective observation matrix:

$$H(k) = \left[H_1^T(k), \ldots, H_N^T(k)\right]^T \ , \tag{2}$$

and as an effective additive measurement noise:

$$v(k) = \left[v_1^T(k), \ldots, v_N^T(k)\right]^T \ , \tag{3}$$

with the further assumption that the zero mean white Gaussian measurement noises across partitions (i.e., between sensors as a consequence between different planar views for the 2-D generalizations to come) are uncorrelated (from sensor to sensor) so that the associated covariance intensity matrices are of the form

$$E\left[v(k)v^T(k)\right] = \text{blockdiagonal}\{R_1, \ldots, R_N\} \ . \tag{4}$$

Similarly, let each sensor's local system model consist of the same $n \times 1$ state vector in common throughout, of the form

$$x_i(k+1) = \Phi_i(k+1,k)x_i(k) + w_i(k) \ , \tag{5}$$

with a suitably tailored (specialized) $m_i \times 1$ vector measurement model for sensor i of the form

$$z_i(k) = H_i(k)x_i(k) + v_i(k) \ , \tag{6}$$

so each local filter, using the alternate KF formulation, is expressible as in Table 2.

[3] The distinction being in how the Kalman gain is calculated (cf., Tables 1 and 2).

[4] All reminiscent of the earlier structural result of [71].

[5] It is unlikely that an analyst will be able to block partition both of two independent quantities (i.e., states, x, and measurements, z), as is necessary within the framework of the methodology espoused in [55], and expect to get the same number of blocks, each corresponding sub-block being of the same size. Parallel filtering applications seldom have state partitions that are not redundant, a structure that is apparently required in the architecture and methodology of [55] and therefore could be perceived to be a severe limitation.

[6] While delays incurred in packet switching or other types of network communication delays are not explicitly recognized by the methodology offered in [57], it may still be implicitly handled since mis-synchronization and delay are frequently modeled as just noisy versions of the otherwise ideal quantities, as a standard buy-off used to acknowledge and compensate for the presence of data senescence. Here perturbational uncertainty in the registration time is instead replaced by perturbational uncertainty in the values of the quantities under scrutiny at a presumed exact time step.

	PROPAGATE STEP	UPDATE STEP
COVARIANCE	$P_i(k\|k-1) = \Phi_i(k,k-1)P_i(k-1\|k-1)\Phi_i^T(k,k-1) + Q_i(k)$	$P_i(k\|k) = [P_i^{-1}(k\|k-1) + H_i^T(k)R_i^{-1}(k)H_i(k)]^{-1}$
FILTER GAIN		$K_i(k) = P_i(k\|k)H_i^T(k)R_i^{-1}$
FILTER	$\hat{x}_i(k\|k-1) = \Phi_i(k,k-1)\hat{x}_i(k-1\|k-1)$	$\hat{x}_i(k\|k) = \hat{x}_i(k\|k-1) + K_i(k)(z_i(k) - H_i(k)\hat{x}_i(k\|k-1))$

Table 2: "Alternate" Kalman Filter Implementation/Mechanization Equations (as separate local filters not yet collated/coordinated/combined)

The vehicle or contrivance for linking up these results for eventual decentralized filtering is the formation of the centralized $H^T(k)R^{-1}(k)H(k)$ as

$$H^T(k)R^{-1}(k)H(k) = \sum_{j=1}^{N} H_j^T(k)R_j^{-1}(k)H_j(k) . \tag{7}$$

Now from Table 2, the covariance update formula for the i^{th} sensor may be rewritten as

$$P_i^{-1}(k|k) - P_i^{-1}(k|k-1) = \underbrace{H_i^T(k)R_i^{-1}(k)H_i(k)}_{\text{LHS info broadcast from each local sensor } i \text{ is equivalent to this covariance correction info,}} \tag{8}$$

and similarly for the aggregate global centralized covariance update as

$$P^{-1}(k|k) - P^{-1}(k|k-1) = \sum_{j=1}^{N} H_j^T(k)R_j^{-1}(k)H_j(k) , \tag{9}$$

which may now be reexpressed (by substituting Eq. 8 in Eq. 9) as

$$P^{-1}(k|k) - P^{-1}(k|k-1) = \sum_{j=1}^{N} \left[P_j^{-1}(k|k) - P_j^{-1}(k|k-1)\right] , \tag{10}$$

as an equation for the global covariance update in terms of the summation of local entities (consisting of $n(n+1)/2 + n = n(n+3)/2$ floating point variables) originally calculated at the j^{th} sensor (j=1 to N) and broadcast via a communication network to a processor node that is tasked with collating all the local information into a global *best answer*.

Another benefit of block decomposition of the aggregate centralized form is in exposing the following equivalence that exists:

$$H^T(k)R^{-1}(k)z(k) = \sum_{j=1}^{N} H_j^T(k)R_j^{-1}(k)z_j(k) . \tag{11}$$

Another simplifying contrivance is the observation from the covariance update, known as Joseph's form, which is known to be mathematically equivalent to

$$P(k|k) = [I - K(k)H(k)]\, P(k|k-1) \tag{12}$$

(cf., [54, Eq. 20], [55, Eq. 9], [57, Eq. B11]). Then post-multiplying throughout Eq. 12 above by $P^{-1}(k|k-1)$ yields

$$[I - K(k)H(k)] = P(k|k)P^{-1}(k|k-1) . \tag{13}$$

Now by taking the estimate update equation as

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)(z(k) - H(k)\hat{x}(k|k-1)) , \tag{14}$$

we further have that

$$\begin{aligned}\hat{\mathrm{x}}(\mathrm{k}|\mathrm{k}) &= \hat{x}(k|k-1) + K(k)(z(k) - H(k)\hat{x}(k|k-1)) \\ &= [I - K(k)H(k)]\,\hat{x}(k|k-1) + P(k|k)H^T(k)R^{-1}(k)z(k) \end{aligned} \tag{15}$$

and substituting for $[I - K(k)H(k)]$ from Eq. 13 and pre-multiplying throughout by $P^{-1}(k|k)$ yields

$$\begin{aligned}\mathrm{P}^{-1}(\mathrm{k}|\mathrm{k})\hat{\mathrm{x}}(\mathrm{k}|\mathrm{k}) &= P^{-1}(k|k)P(k|k)P^{-1}(k|k-1)\hat{x}(k|k-1) + P^{-1}(k|k)K(k)z(k) \\ &= P^{-1}(k|k-1)\hat{x}(k|k-1) + P^{-1}(k|k)P(k|k)H^T(k)R^{-1}(k)z(k) \\ &= P^{-1}(k|k-1)\hat{x}(k|k-1) + H^T(k)R^{-1}(k)z(k) \\ &= P^{-1}(k|k-1)\hat{x}(k|k-1) + \sum_{j=1}^{N} H_j^T(k)R_j^{-1}(k)z_j(k)\,, \end{aligned} \tag{16}$$

and, by now pre-multiplying Eq. 16 throughout by $P(k|k)$, yields the fundamental estimation update expression:

$$\hat{\mathrm{x}}(\mathrm{k}|\mathrm{k}) = P(k|k)\left[P^{-1}(k|k-1)\hat{x}(k|k-1) + \sum_{j=1}^{N} H_j^T(k)R_j^{-1}(k)z_j(k)\right]. \tag{17}$$

as an equation for the global state update in terms of the summation of local entities originally calculated at the i^{th} sensor (i=1 to N) and broadcast via a communication network to a processor node that is tasked with collating all the local information into a global best answer.

By a derivation route and arguments identical to that presented for Eqs. 11 to 17, we obtain a local state estimation equation of a form similar to that of Eq. 17 for each local sensor as

$$\hat{\mathrm{x}}_{\mathrm{i}}(\mathrm{k}|\mathrm{k}) = P_i(k|k)\left[P_i^{-1}(k|k-1)\hat{x}_i(k|k-1) + H_i^T(k)R_i^{-1}(k)z_i(k)\right], \tag{18}$$

or, rearranged to be

$$\mathrm{P}_{\mathrm{i}}^{-1}(\mathrm{k}|\mathrm{k})\hat{\mathrm{x}}_{\mathrm{i}}(\mathrm{k}|\mathrm{k}) - \mathrm{P}_{\mathrm{i}}^{-1}(\mathrm{k}|\mathrm{k}-1)\hat{\mathrm{x}}_{\mathrm{i}}(\mathrm{k}|\mathrm{k}-1) = \underbrace{H_i^T(k)R_i^{-1}(k)z_i(k)}_{\text{LHS info broadcast from each local sensor } i \text{ is equivalent to this state estimate correction info.}} \tag{19}$$

In conclusion, the final architecture for centralized globally optimal estimates obtainable from the indicated info broadcast on the network from each local sensor i is derivable from Eq. 19 substituted into Eq. 17 as

$$\hat{\mathrm{x}}(\mathrm{k}|\mathrm{k}) = \mathrm{P}(\mathrm{k}|\mathrm{k})\left[\mathrm{P}^{-1}(\mathrm{k}|\mathrm{k}-1)\hat{\mathrm{x}}(\mathrm{k}|\mathrm{k}-1) + \sum_{\mathrm{j}=1}^{\mathrm{N}}\{\mathrm{P}_{\mathrm{j}}^{-1}(\mathrm{k}|\mathrm{k})\hat{\mathrm{x}}_{\mathrm{j}}(\mathrm{k}|\mathrm{k}) - \mathrm{P}_{\mathrm{j}}^{-1}(\mathrm{k}|\mathrm{k}-1)\hat{\mathrm{x}}_{\mathrm{j}}(\mathrm{k}|\mathrm{k}-1)\}\right], \tag{20}$$

(cf., [54, Eq. 26], [55, Eq. 16b], [57, Eq. 51]) to be used along with the covariance update of Eq. 10, rearranged as

$$P(k|k) = \left[P^{-1}(k|k-1) + \left[\sum_{j=1}^{N}\{P_j^{-1}(k|k) - P_j^{-1}(k|k-1)\}\right]^{-1}\right]^{-1} = \left[A^{-1} + B^{-1}\right]^{-1} = A\left[A+B\right]^{-1}B\,, \tag{21}$$

(cf., [54, Eq. between Eqs. 17 and 18], [55, Eq. 17b], [57, Eq. 52]) [7] which is recognized to be of the form of a triple $n \times n$ matrix inversion, where operations counts for each of these inversions is merely n^3 (or more exactingly $O(n^{2.71})$

[7]Notice that there are slight discrepancies between what is summarized here and what was offered at comparable steps in [57] so, strictly speaking, the approach of [57] is not *identical* to that of [54] and [55] even if *most* of the particulars are the same. Similarly, [55] looks further into an information filter formulation and a square root filter formulation after it has passed through these same primary results that are revealed here to be in common with the other two approaches. However, these further formulations are rather routine KF variations.

according to Ronald Rivest at MIT and others). Reiterating, reference [54] recommends that each local sensor node i broadcast two pieces of critical information at each designated synchronous time step k being (1) the $n \times n$ matrix difference $\{P_i^{-1}(k|k) - P_i^{-1}(k|k-1)\}$ and (2) the $n \times 1$ vector difference $\{P_i^{-1}(k|k)\hat{x}_i(k|k) - P_i^{-1}(k|k-1)\hat{x}_i(k|k-1)\}$, which have now already been demonstrated above to be equivalent to transmitting the normally expected natural info on $H_i(k)$, $R_i^{-1}(k)$, and $z_i(k)$ (respectively, of dimension $m_i \times n$, $m_i \times m_i$, and $m_i \times 1$). However, since the above matrix difference arising in Eq. 21 is symmetric, one only needs to actually transmit $n(n+1)/2$ entries of the matrix rather than n^2 at each time step k as a considerable savings. The obvious perceived benefit of the above formulation is *structural consistency* (independence of particular local m_i) in the collating update architecture of Eqs. 20 and 21. Although, unstated in [54]-[57], an even greater perceived benefit of the architecture being offered is that for sensors that fail to report by the designated collation time for time-step k (due to possible failures, battle damage, overly delayed message packets, pruned outlier readings, etc.) the summation can still take place (using info from those sensors that do report) to yield *the best there is* with the collection of local information available at the time!

Please notice that the more recent investigations [23], [51], [58], [59], and [68, p. 298] all favorably reference the present author's earlier work of [10]. Besides currently pursuing in-house numerical evaluation of the utility of more recently refined decentralized filter design originally put forth as a less refined version in [10, Sec. IV.C, Fig. 8], it is helpful to be aware that other researchers are independently evaluating this design and critically comparing it to alternate implementation approaches in also comparing it to [52] and [57] that [58] and [50] evidently missed [perhaps because it was so recent then]). A multitude of independent assessments hopefully makes for a healthier (eventually) unbiased final tally [8].

This author had previously cautioned (or reminded) the estimation community in [67, p. 944, Eq. 47] not to make the <u>mistake</u> of using the simpler version of the discrete-time Kalman Covariance Update Equation:

$$P_{k|k} = [I - K_k H_k] P_{k|k-1} , \tag{22}$$

when the following (so-designated Joseph's form) should be used instead [9]:

$$P_{k|k} = [I - K_k H_k] P_{k|k-1} [I - K_k H_k]^T + K_k R K_k^T . \tag{23}$$

However, Ren Da did include this unfortunate oversight in the preliminary review version of [23]. Da was also pursuing the reduced-order filtering problem in [23] but, unfortunately, so many of the existing *so-called reduced-order filtering methodologies* currently being employed are flawed, with enumerations detailed in [26, pp. 79-82]. The author Da had some interesting new ideas on use of optimal and simpler sub-optimal "combining rules" (for combining the local estimation results from separate local filters to obtain the globally optimal estimate as an outcome) that are of interest in what is discussed later. The present author has also obtained sub-optimal more expedient combining rules in [8, Sec. 1.5].

2.2 Status of 2-D Kalman filtering

Generalizations of standard 1-D random process evolving in time or indexed on a single time variable (isomorphic to the real line so that it is *totally ordered* for simply distinguishing past from present from future [i.e., for any t_1 and t_2, either $t_1 < t_2$, or $t_1 = t_2$, or $t_1 > t_2$] and having a standard unique definition of causality) have already been extended to 2-D [60] for Input/Output realizations. Early approaches to 2-D modeling usually invoked non-symmetric half-plane (NSHP) type causality merely for simplicity and convenience [30], [65].

The following representative milestones are recounted in briefly summarizing the generalization of Kalman filter formulations from 1-D to 2-D:

- Although Eugene Wong [13] alerts the reader in the mid 1970's and raises their level of consciousness to appreciate the difficulty of this problem (since the 2-D planar index of a random field can't be *totally ordered* for a clear unambiguous delineation of what's past, present, and future as can be done for the real line [as occurs for the time index of a random process]; however, the 2-D plane can be *partially ordered* but partial orderings are not unique and are also not wholly satisfying since there are several viable candidates that are reasonable

[8] A critique of Carlson's decentralized filtering approach [50] appeared in the November 1991 issue of IEEE Trans. on Aerospace and Electronic Systems as [12].

[9] The former expression doesn't yield the correct covariance associated with using a reduced-order suboptimal Kalman gain K_k while the later expression does.

to use but all have ambiguous "past", "present" (being a set rather than being a mere point, as occurs with a random process), and "future" defined, depending on which partial ordering convention is invoked). While [13] originally doesn't extend much hope for immediate resolution, a few years later he reports substantial progress in this area [14], [15] [10].

- In the 1980's, Howard Kaufman along with his students and colleagues blazed an impressive development trail in further generalizations of 2-D Kalman filters specifically for image restoration applications [16]–[20]. In particular:
  - Quoting [16]: "it is established that for typical autoregressive signal models with nonsymmetric half-plane support, the dimension of the state size to be used within the Kalman filter is approximately equal to the product of the image model order and the pixel width of the image."
  - Quoting [19]: " a parallel identification and restoration procedure is described for images with symmetric noncausal blurs. It is shown that the identification problem can be recast as a parallel set of one dimensional ARMA identification problems. By expressing the ARMA models as equivalent infinite-order AR models **(sic) [the present TeK Associates' author takes issue with this limiting claim and clarifies why in the first bullet in Sec. 2.3]**, an entirely linear estimation procedure can be followed."
  - Quoting [20]: "it is established that an EKF for on-line parameter Identification was found to be unsuitable for blur parameter identification **(sic) [the present TeK Associates' author takes issue with this limiting claim and clarifies why in the second bullet in Sec. 2.3]** because of the presence of significant process noise terms that caused large deviations between the predicted pixel estimates and the true pixel intensities."
  - Quoting [18]: "model-based segmentation and restoration of images is performed. It was assumed that space-variant blur can be adequately represented by a collection of L distinct point-spread functions, where L is a predefined integer. (*The 'Multiple Model of Magill' (MMM)*) bank of parallel Kalman filters was applied to this problem." **See Sec. 3 for more about MMM.**
  - Quoting [20]: "it is revealed that image restoration based upon unrealistic homogeneous image and blur models can result in highly inaccurate estimates with excessive ringing. Thus it is important at each pixel location to restore the image using the particular image and blur parameters characteristic of the immediate local neighborhood."

2.3 Correcting a few analytical misconceptions occurring within the historical principal paths summarized above

The following two items are offered as corrections to aspects raised in Sec. 2.2 regarding use of 2-D Kalman filters for image restoration and consequently should improve performance in the application activity reported above:

- From [40, Eqs. 23, 24], the following 2-input/2-output continuous-time process:

$$\ddot{x}_1 + 3\,\dot{x}_1 + 2\,x_1 = -\dot{u}_1 - \sqrt{7}/2\;u_1 - (1/2)\,u_2, \tag{24}$$

$$\ddot{x}_2 + 3\,\dot{x}_2 + 2\,x_2 = -\dot{u}_1 - \sqrt{7}/2\;u_1 + (3/2)\,u_2\;, \tag{25}$$

after rearranging, is recognized to indeed be of the ARMA form $\sum_{p=0}^{N} A_p \underline{x}^{(p)}(t) = \sum_{q=0}^{M} B_q \underline{u}^{(q)}(t)$:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + \begin{bmatrix} -\sqrt{7}/2 & -1/2 \\ -\sqrt{7}/2 & 3/2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{26}$$

and the following 2-input/4-intermediate output/2-output continuous-time process from [40, Eqs. 34, 35]:

$$\begin{bmatrix} \dot{x}_1' \\ \dot{x}_2' \\ \dot{x}_3' \\ \dot{x}_4' \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ (6-\sqrt{7})/2 & -1/2 \\ -1 & 0 \\ (6-\sqrt{7})/2 & 3/2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{27}$$

[10] Incidently, he also went from department head at U.C. Berkeley to *Science Advisor* in the Bush Administration.

with

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \triangleq \begin{bmatrix} x_1' \\ x_3' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix}, \tag{28}$$

can be recognized to be the AR form $\sum_{p=0}^{N} A_p \underline{x}^{(p)}(t) = B_0 \underline{u}(t)$ (via a common matrix pre-multiplaction throughout) from:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1' \\ \dot{x}_2' \\ \dot{x}_3' \\ \dot{x}_4' \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{29}$$

yet both have the same identical transfer function matrix (demonstrated in [41]), namely:

$$W^T(s) = \begin{bmatrix} \frac{-s-(\sqrt{7}/2)}{(2+s)(1+s)} & \frac{-s-(\sqrt{7}/2)}{(2+s)(1+s)} \\ \frac{-1/2}{(2+s)(1+s)} & \frac{3/2}{(2+s)(1+s)} \end{bmatrix}^T . \tag{30}$$

and so are equivalent systems since both representations represent observable and controllable portions (with a straightforward change in notation of the variable names occurring in [40] to what is used here to make the ARMA and AR forms more apparent). Notice that the ARMA process representation has a finite number of terms and so does the equivalent AR process representation (unlike what was asserted in [19]).

- Use of Extended Kalman Filters for parameter estimation in [20] used only the *standard form* of EKF. Ljung has shown [70] that in order to obtain consistent parameter estimation results from an EKF (that are comparable in performance to other more specialized parameter identification techniques such as recursive least squares estimators (RLE) or Instrumental Variables), an additional term must be included in the EKF that usually doesn't appear there in other more standard nonlinear estimation applications of this tool (such as in "target tracking") where linearization simplifications are invoked without contaminating the objectives. Additionally, use of re-linearization usually improves the performance of an EKF (by improving the efficacy of linearization) for just a modest increase in algorithm complexity [39].

3 ADAPTIVE CONTROL-LIKE IMAGE PROCESSING

For random field processing to run autonomously without human intervention or man-in-the-loop to reset or adjust parameters when underlying conditions change, there is a need for adaptive schemes to assess the prevailing conditions and correct for them in the processing. The techniques of the last decade and a half for performing similar tasks within the corresponding two stages of adaptive control [11], that similarly must first have unknown or varying parameters identified by some scheme, appear to be a hodge-podge of different approaches, but [62] has recently appeared on the scene to unify and demonstrate how these alternate approaches can still lead to the same result and how a proper understanding of it all can be used to generate new results, as needed in certain areas of adaptive processing such as we face now.

As a somewhat less desirable alternate backup route to exclusive reliance on just multi-filter combining rules, consider combining sensors using a multiple model bank-of-Kalman-filters (as originally developed by D. T. Magill in 1965, clarified and explained by Demitri Laniotis in 1967, and simplified in its specialized implementation by R. Grover Brown (Iowa State-Ames) in 1981, and further utilized in this simplified form by AFIT's Peter Maybeck in another context for IR tracking in modifications evolving from 1983 to present). Each filter in the bank could have a different hypothesized noise intensity level to provide a degree of adaptivity to underlying noise conditions

[11] I.e., it is well known that "estimation" and "control" are mathematical duals in that the same descriptive equations apply, while physical motivations and/or justifications differ.

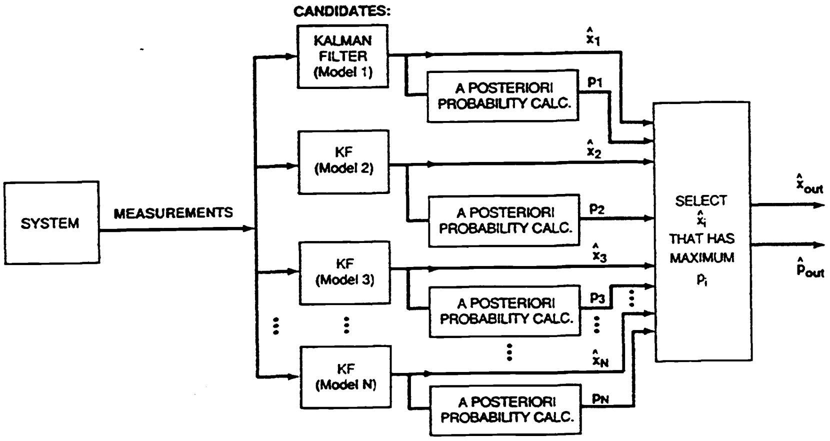

Figure 5: Multiple Model of Magill (MMM): N alternative Kalman filters, each with its distinctly different system models vying to match the true system as it progresses through its likely operating regimes, with associated on-line computation of probabilities of each being correct so that a tally is available to decide which one (choice of a "winner" varying with time) offers the best match.

or quantized blur parameter variations (e.g., [63] [12], [64]). [**TeK Associates** found that limited success along these lines has in fact already been achieved [18].]

This so-called Multiple Model of Magill (MMM) consists of a parallel bank-of-Kalman-filters, with associated on-line calculation of probabilities [13] of the best match to the actual system under test, as depicted in Fig. 5. However, until the last decade, MMM represented too large a computational burden to be embraced for most practical applications. However, that situation has now changed with the advent of parallel processing, cheaper RISC implementations, and ASIC/VLSI/VHSIC, DEC's Alpha, and Intel Pentium chips, using Microsoft's $Windows^{NT}$, IBM's OS/2, and Microsoft's *Windows* 95.

Another lucrative approach (endorsed in [17, p. 19, col. 2] as a valid alternative for simultaneous Image and blur identification) is spelled out here. According to [17], another approach to image restoration that can be pursued would be to use aggregate ARMA modeling to capture the salient modeling aspects (without any need for the detailed physical basis of the interconnections being spelled out, as is otherwise normally done in a Kalman filter-based state space model). If we were to further pursue taking just a phenomenological ARMA approach without regard for underlying state identities, then the simplifications availed from [40] in now recognizing some connections or exact correspondence between strict ARMA modeling (a harder nonlinear problem) and strict AR modeling (an easier linear problem [but of a slightly higher but still finite dimension in order to correspond exactly to what an ARMA model would capture, as long as the AR portion is of higher order than the MA portion]). Recent simplifications are now available (i.e., [66]) for implementing realizations of a linear system representation by using the numerically stable and now popular Singular Value Decomposition (SVD) for computations. Use of Expectation-Maximization or E-M algorithms [61] are also hailed as reasonable alternative paths to use in further pursuing specification of a few unknown critical parameters that are needed to really get going in using the ARMA approach ("since the E-M algorithm essentially solves two linear sub-problems on each iteration").

[12] Yaakov Bar-Shalom and Hank Blom also use a generalization of MMM (denoted as IMM) and have a nice description of the accompanying probability calculations of IMM which, in turn, determines which running filter model most closely corresponds to the actual measurements received.

[13] Initially, each candidate filter is equally likely. Engineering tuning can also be done on these probability calculations (such as putting upper and lower limit stops so that answers are never reached with total certainty) therefore the MMM stays open minded for changes in system operational regime and correspondingly to the best filter hypothesis that matches it.

4 TeK Associates'EXTENSIONS FOR SENSOR FUSION

We have pursued the idea that recent "decentralized" Kalman filter (KF) technology [6]–[12], by outfitting each participating imaging sensor with its own dedicated 2-D Kalman filter [14] can be used as the basis of a sensor fusion methodology that allows a final collating filter to assemble the data from diverse imaging sensors of various resolutions into a single resulting image that combines all the available information [10], [23] (in analogy to what is already routinely done in multisensor NAVIGATION [10], [24]–[27]). The novelty is in working out the theoretical details for 2-D filtering situations (using [28]–[31] as a guide) while assuming that the image registration problem (reduction to a common scale and coordinated alignment registration) has already been independently handled [32], perhaps by hardware proximity multiplexing through a shared common aperture [perhaps using rotating mirrors] where scale of sensor scene image could have been calibrated and adjusted in a static environment beforehand). We must synchronize frame size and location of pixels of interest to be comparably located with same "raster scan" speed and size used for each to match up for different sensors. Rule for Kalman filters is that the combining of underlying measurements or sensor information can only help and never hurt. We interpret this approach as involving several common views of the same scene, as instantaneously obtained from different sensors, all stacked up vertically one planar view on top of another planar view, each with its own local 2-D Kalman-like image restoration filter proceeding to raster scan (in multi-layer sync). Then apply the multi-filter combining rules from Decentralized filtering to the bunch to obtain a single best estimate image as the resulting output as a convenient and useful methodology to achieve sensor fusion.

While we have sketched out details of the constituents of **TeK Associates**' approach to image restoration and sensor fusion and its rationale, we are reluctant to just magnanimously divulge the fruits of our IRD labors, namely, our particular 2-D mechanization equations at this time since we view these as being **TeK** proprietary (at least until we issue this as a product upgrade to $\mathbf{TK-MIP}^{TM}$). We obtained encouraging results from preliminary simulations performed with the **MatLab** Image Processing Toolbox, but don't bother to show how it enhanced "Lena's" portrait here [15].

Image fusion applications exist in machine vision and in medicine (ultrasound, x-ray, NMR/NMI) as well as in military Surveillance/Reconnaissance (Lidar, millimeter wave radar imagery, IR, UV, TV). However, our immediate interest is to introduce this as a future capability into **TeK Associates**' current software product for PC's: $\mathbf{TK-MIP}^{TM}$ in extending it to **WindowsNT** or any other comparable Operating System that supports preemptive multi-processing on parallel add-in processor boards (one for each sensor filter and one dedicated exclusively for the final combining rule for the end result).

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[15] One technical problem that we did encounter was with **MatLab**'s new capability to isolate level-crossing instant of either constant or specified time-varying thresholds with almost infinite precision. This is true only for completely deterministic situations since the underlying algorithms are predictor/corrector which are stymied when noise [albeit pseudo-random noise (PRN)] is introduced in the simulation. The presence of noise has been the bane of all but the crudest of integration methodologies since the earliest days of digital simulation. However, engineering applications where threshold comparisons are crucial usually include the presence of noise as in detection (i.e., is the desired signal present or just noise) in radar or communications, in Kalman filter-based failure detection or maneuver detection [47], or in peak picking as it arises in sonar processing [38] and in image processing [69].

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