

My Comments that previously appeared at the bottom of The MathWorks' Blog on Kalman Filters

23 Nov 2017

Thomas H. Kerr III, now d.b.a. as TeK Associates, was involved with D-1 Trident and C-4 back-fit Poseidon SSBN SINS/ESGM navigation development from 1973-1979 (where I developed an original automated failure detection algorithm for the ESGM as a Kalman filter real-time accouterment involving Two Confidence Regions:

1. Kerr, T. H., "Failure Detection Aids for Human Operator Decisions in a Precision Inertial Navigation System Complex," Proceedings of Symposium on Applications of Decision Theory to Problems of Diagnosis and Repair, Keith Womer (editor), Wright-Patterson AFB, OH: AFIT TR 76-15, AFIT/EN, Oct. 1976, sponsored by Dayton Chapter of the American Statistical Association, Fairborn, Ohio, pp. 98-127, June 1976.
2. Kerr, T. H., "Real-Time Failure Detection: A Static Nonlinear Optimization Problem that Yields a Two Ellipsoid Overlap Test," Journal of Optimization Theory and Applications, Vol. 22, No. 4, pp. 509-535, August 1977.
3. Kerr, T. H., "Statistical Analysis of a Two Ellipsoid Overlap Test for Real-Time Failure Detection," IEEE Transactions on Automatic Control, Vol. 25, No. 4, pp. 762-773, August 1980.
4. Kerr, T. H., "False Alarm and Correct Detection Probabilities Over a Time Interval for Restricted Classes of Failure Detection Algorithms," IEEE Transactions on Information Theory, Vol. 28, No. 4, pp. 619-631, July 1982.
5. Kerr, T. H., "Examining the Controversy Over the Acceptability of SPRT and GLR Techniques and Other Loose Ends in Failure Detection," Proceedings of the American Control Conference, San Francisco, CA, pp. 966-977, 22-24 June 1983. (an expose)
6. Kerr, T. H., "Comments on 'A Chi-Square Test for Fault Detection in Kalman Filters'," IEEE Transactions on Automatic Control, Vol. 35, No. 11, pp. 1277-1278, November 1990.
7. Kerr, T. H., "A Critique of Several Failure Detection Approaches for Navigation Systems," IEEE Transactions on Automatic Control, Vol. 34, No. 7, pp. 791-792, July 1989. (an expose)
8. Kerr, T. H., "On Duality Between Failure Detection and Radar/Optical Maneuver Detection," IEEE Transactions on Aerospace and Electronic Systems, Vol. 25, No. 4, pp. 581-583, July 1989.
9. Kerr, T. H., "Comments on 'An Algorithm for Real-Time Failure Detection in Kalman Filters'," IEEE Trans. on Automatic Control, Vol. 43, No. 5, pp. 682-683, May 1998. (an expose of sorts)
10. Kerr, T. H., "Integral Evaluation Enabling Performance Trade-offs for Two Confidence Region-Based Failure Detection," AIAA Journal of Guidance, Control, and Dynamics, Vol. 29, No. 3, pp. 757-762, May-Jun. 2006.

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I posed and solved the **problem of SSBN external navaid fix utilization while evading enemy surveillance as a "cat-and-mouse" game of "sensor schedule optimization" within a natural Kalman Navigation filter context:** Publications constituting significant development in submarine navigation trade-off considerations between frequency of external navaid usage (to maintain sufficient navigation accuracy in case a launch is ordered) versus exposure to enemy surveillance:

1. Kerr, T. H., "Preliminary Quantitative Evaluation of Accuracy/Observables Trade-off in Selecting Loran/NAVSAT Fix Strategies," TASC Technical Information Memorandum TIM-889-3-1, Reading, MA, December 1977 (Confidential).
2. Kerr, T. H., "Improving C-3 SSBN Navaid Utilization," TASC TIM-1390-3-1, Reading, MA, August 1979 (Secret).
3. Kerr, T. H., "Modeling and Evaluating an Empirical INS Difference Monitoring Procedure Used to Sequence SSBN Navaid Fixes," Proceedings of the Annual Meeting of the Institute of Navigation, U.S. Naval Academy, Annapolis, Md., 9-11 June 1981 (reprinted in Navigation: Journal of the Institute of Navigation, Vol. 28, No. 4, pp. 263-285, Winter 1981-82).
4. Kerr, T. H., "Sensor Scheduling in Kalman Filters: Evaluating a Procedure for Varying Submarine Nav aids," Proceedings of 57th Annual Meeting of the Institute of Navigation, pp. 310-324, Albuquerque, NM, 9-13 June 2001 (an update).

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Results for several other diverse Kalman filter applications relating to use of Kalman-like filters for Early Warning Radars to track enemy RV's: 1. Kerr, T. H., "Angle-Only Tracking," slide presentation for Reentry Systems Program Review, Lincoln Laboratory, Lexington, MA, 10 Jan. 1989. 2. Kerr, T. H., "An Analytic Example of a Schwappe Likelihood Ratio Detector," IEEE Transactions on Aerospace and Electronic Systems, Vol. 25, No. 4, pp. 545-558, Jul. 1989. (closed-form analytic solution) 3. Kerr, T. H., "Streamlining Measurement Iteration for EKF Target Tracking," IEEE Transactions on Aerospace and Electronic Systems, Vol. 27, No. 2, pp. 408-421, Mar. 1991. 4. Kerr, T. H., "Use of GPS/INS in the Design of Airborne Multisensor Data Collection Missions (for Tuning NN-based ATR algorithms)," the Institute of Navigation Proceedings of GPS-94, Salt Lake City, pp. 1173-1188, 20-23 Sep. 1994. 5. Kerr, T. H., "Assessing and Improving the Status of Existing Angle-Only Tracking (AOT) Results," Proceedings of the International Conference on Signal Processing Applications & Technology, Boston, pp. 1574-1587, 24-26 Oct. 1995. (an expose) 6. Kerr, T. H., "Extending Decentralized Kalman Filtering (KF) to 2-D for Real-Time Multisensor Image Fusion and/or Restoration," Proceedings of SPIE Conference, Vol. 2755, Orlando, pp. 548-564, 8-10 Apr. 1996. 7. Kerr, T. H., "Extending Decentralized Kalman Filtering (KF) to 2D for Real-Time Multisensor Image Fusion and/or Restoration: Optimality of Some Decentralized KF Architectures," Proceedings of the International Conference on Signal Processing Applications; Technology, Boston, MA, pp. 155-170, 7-10 Oct. 1996. **A related Cramer Rao Lower Bound (CRLB) computational methodology for evaluating and gauging the efficacy of using Kalman-filter-like "Extended Kalman filters" for nonlinear estimation applications:** 8. Kerr, T. H., "Developing Cramer-Rao Lower Bounds to Gauge the Effectiveness of UEWR Target Tracking Filters," Proceedings of AIAA/BMDO Technology Readiness Conference and Exhibit, Colorado Springs, 3-7 August 1998. 9. Satz, H. S., Kerr, T. H., "Comparison of Batch and Kalman Filtering for Radar Tracking," Proceedings of 10th Annual AIAA/BMDO Conference, Williamsburg, VA, 25 Jul. 2001 (Unclassified, but Conference Proceedings are SECRET).

24 Nov 2017

Publications that combine the ideas of failure detection with those of decentralized Kalman Filtering to yield a rigorous basis for system reconfiguration and redundancy management: (1) Kerr, T. H., "Decentralized Filtering and Redundancy Management Failure Detection for Multi-Sensor Integrated Navigation Systems," Proceedings of the National Technical Meeting of the Institute of Navigation (ION), San Diego, CA, 15-17 January 1985. (an expose) (2) Kerr, T. H., "Decentralized Filtering and Redundancy Management for Multisensor Navigation," IEEE Trans. on Aerospace and Electronic Systems, Vol.23, No. 1, pp. 83-119, Jan. 1987. (an expose)

25 Nov 2017

Matching Up with Image Integrity approach to "Observability" (Cont.'d #2): Rigorous MKF updates in airborne estimation for attitude determination (Cont.'d): (15) Choukroun, D., Weiss, H., Bar-Itzhack, I. Y., Oshman, "Kalman Filtering for Matrix Estimation," IEEE Trans. on Aerospace and Electronic Systems, Vol. 42, No. 1, pp. 147-159, Jan. 2006: **A linear Matrix Kalman filter for DCM. DCM (Refinement #1)** (16) Choukroun, D., Weiss, H., Bar-Itzhack, I. Y., Oshman, "Direction Cosine Matrix Estimation from Vector Observations Using a Matrix Kalman Filter," AIAA Guidance, Navigation, and Control Conference and Exhibit, pp. 1-11, Aug. 2003: **A linear Matrix Kalman Filter for DMC using either vector or matrix measurement updates. DCM Refinement #2.** (17) Choukroun, D., "A Novel Quaternion Kalman Filter using

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GPS Measurements," Proceedings of ION GPS, Portland, OR, pp. 1117-1128, 24-27 Sep. 2002: **An alternative viewpoint. (Quaternion Refinement #1.)** (18) Choukroun, D., Weiss, H., Bar-Itzhack, I. Y., Oshman, "Kalman Filtering for Matrix Estimation," IEEE Trans. on Aerospace and Electronic Systems, Vol. 42, No. 1, pp. 147-159, Jan. 2006: **Quaternion Refinement #2.** (19) Choukroun, D., Bar-Itzhack, I. Y., Oshman, "Novel Quaternion Kalman Filter," IEEE Trans. on Aerospace and Electronic Systems, Vol. 42, No. 1, pp. 174-190, Jan. 2006: **Quaternion Refinement #3.** (20) Choukroun, D., Weiss, H., Bar-Itzhack, I. Y., Oshman, "Direction Cosine Matrix Estimation From Vector Observations Using A Matrix Kalman Filter," Proceedings of AIAA Guidance, Navigation, and Control Conference and Exhibit, Austin, TX, pp. 1-11, 11-14 August 2003: **DCM Refinement #3** (21) Choukroun, D., "Ito Stochastic Modeling for Attitude Quaternion Filtering," Proceedings of Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference, Shanghai, P. R. China, pp. 733-738, 16-18 Dec. 2009: **Quaternion Refinement #4.** Matrix KF material that we seek to exploit was **primarily developed by Daniel Choukroun**, B. S. (Summa cum Laude), M.S., Ph.D. (1997, 2000, 2003), post-doc (UCLA), currently (~2012) Assistant Professor at Delft University of Technology, Netherlands. **Needs to utilize two different sensors with simultaneous views of objects in the landscape in order to deduce angles! NASA updates in Spaceborne estimation for attitude determination:** (22) Cheng, Y., Landis Markley, F., Crassidis, J. L. Oshman, Y., "Averaging Quaternions," Advances in the Astronautical Sciences series, Vol. 127, American Astronautical Society, AAS paper No. 07-213, 2007. (23) Landis Markley, F., "Attitude Filtering on SO(3)," Advances in the Astronautical Sciences series, Vol. 122, American Astronautical Society, AAS paper No. 06-460, 2006. (24) Cheng, Y., Crassidis, J. L., and Landis Markley, F., "Attitude Estimation for Large Field-of-View Sensors," Advances in the Astronautical Sciences series, Vol. 122, American Astronautical Society, AAS paper No. 06-462, 2006. (25) Landis Markley, F., "Attitude Estimation or Quaternion Estimation?," Advances in the Astronautical Sciences series, Vol. 115, American Astronautical Society, AAS paper No. 03-264, 2003: **Critical and thorough Analysis of 3 different EKF's vs. Technion MKF. MKF was improved.** (26) Reynolds, R., Landis Markley, F., Crassidis, J. L., "Asymptotically Optimal Attitude and Rate Bias Estimation with Guaranteed Convergence," Advances in the Astronautical Sciences series, Vol. 132, American Astronautical Society, AAS paper No. 08-286, 2008. **Estimation Results for Bilinear Systems (to tie into the MKF results):** (27) Halawani, T. U., Mohler, R. R., and Kolodziej, W. J., "A two-step bilinear filtering algorithm," IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. 32, 344-352, 1984. (28) Glielmo, L., Marino, P., Setola, R., Vasca, F., "Parallel Kalman Filter Algorithm for State Estimation in Bilinear Systems," Proceedings of the 33rd Conference on Decision and Control, Lake Buena Vista, FL, pp. 1228-1229, Dec. 1994. (29) Wang, Z., Qiao, H., "Robust Filtering for Bilinear Uncertain Stochastic Discrete-Time Systems," IEEE Trans. on Signal Processing, Vol. 50, No. 3, pp. 560-567, Mar. 2002: **"Robust" approaches usually have sluggish response.** (30) Lopes dos Santos, P., Ramos, J. A., Frias, R., "Derivation of a Bilinear Kalman Filter with Autocorrelated Inputs," Proceedings of the 46th Conference on Decision and Control, New Orleans, LA, pp. 6196-6202, 12-14 Dec. 2007: Structure similar to what Technion **MKF** exhibits, **Matrix Kalman Filter.**

6 Dec 2017

Matching Up with Image Integrity approach to "Observability" (Cont.'d #1): Rigorous updates in airborne estimation for attitude determination: (9) Crassidis, J. L., Markley, F. L., Cheng, Y., "Survey of Nonlinear Attitude Estimation Methods," Journal of Guidance, Control, and Dynamics, Vol. 30, No. 1, pp. 12-28, Jan. 2007: An excellent survey on the subject of attitude estimation. It provides insights into what is important

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in estimation algorithms. It is a more practical and rigorous addendum to their many earlier surveys, concerned with utilizing alternative EKF's or Nonlinear Luenberger Observers (as alternatives to Extended Kalman filter-based approaches). **CONCLUSION: They admonish to "stick with using EKF".** (10) Majji, M., Junkins, J. L., Turner, J. D., "Jth Moment Extended Kalman Filtering for Estimation of Nonlinear Dynamic Systems," AIAA Guidance, Navigation, and Control Conference and Exhibit, Honolulu, HI, Paper No. AIAA 2008-7386, pp. 1-18, 18-21 Aug. 2008: Explores two variations on **JMEKF** formulations that properly handle higher order moments (that lurk in the background while trying to get good estimates and covariances from EKF's). Approximations utilized are acknowledged and properly handled (rather than ignored, as is usually the case). Errors reduced by several orders of magnitude within 5 sec., but results in normalized units (for comparisons to ordinary EKF approach, which it beat by a wide margin). Down side is its larger CPU burden yet to be completely quantified. (11) Scorse, W. T., Crassidis, A. L., "Robust Longitudinal and transverse Rate Gyro Bias Estimation for Precise Pitch and Roll Attitude Estimation in Highly Dynamic Operating Environments Utilizing a Two Dimensional Accelerometer Array," AIAA Atmospheric Flight Mechanics Conference, Paper No. AIAA 2011-6447, Portland, OR, pp. 1-28, 8-11 Aug. 2011: Using the latest in rigorous real-time estimation algorithms (neither a particle filter nor an unscented/Oxford/Sigma-Point filter) for enabling accurate pointing (precise pitch and roll) within an aircraft within a high dynamics operating environment is reported. While it does utilize rate integrating gyros, it also utilizes 2D accelerometer arrays and compares to an onboard gravity map to achieve its accuracy. Following reasonably large offsets, got back to within 0.1 degree pointing error within 10 seconds **but results are much worse with turbulence present.** (12) Jensen, Kenneth J., "Generalized Nonlinear Complementary Attitude Filter," AIAA Journal of Guidance, Control, and Dynamics, Vol. 34, No. 5, pp. 1588-1593, Sept.-Oct. 2011: Achieves a big breakthrough by providing a proof of this particular EKF's global stability as a consequence by stating that it possesses "almost" global asymptotic stability; however, the term "almost" is required terminology to keep probability theorists and purists happy with the wording of his claim. Author Jensen attains his results by utilizing appropriate stochastic Lyapunov functions (proper handling of such is due to Prof. Emeritus Harold J. Kushner, Brown Univ.). (13) La Scala, B. F., Bitmead, R. R., James, M. R., "Conditions for stability of the Extended Kalman Filter and their application to the frequency tracking problem," Math. Control, Signals Syst. (MCSS), vol. 8, No. 1, pp. 1-26, Mar. 1995: **Proof of Stability for yet another EKF. Now worries about EFK divergence evaporate for this application.** (14) Reif, K., Gunther, S., Yaz, E., Unbehauen, R., "Stochastic stability of the continuous-time extended Kalman filter," Proc. Inst. Elect. Eng., Vol. 147, p. 45, 2000: **Proof of Stability for yet another EKF. Now worries about EFK divergence evaporate for this application too.** (14) Salcudean, S., "A globally convergent angular velocity observer for rigid body motion," IEEE Trans. on Autom. Control, Vol. 36, No. 12, pp.1493-1497, Dec. 1991: **Provides proof of Stability for Luenberger Observer use also (~ for EKF).**

9 Dec 2017

Considerations for Simultaneous Localization and Mapping (SLAM): Matching Up with Image Integrity approach to "Observability": (1) Craig Lawson, John F. Raquet, Michael J. Veth, "The Impact of Attitude on Image-Based Integrity," Navigation: Journal of the Institute of Navigation, Vol. 57, No. 4, pp. 249-292, Winter 2010. A summarizing discussion is provided in App. A conveying more details (pp. 15-21) of the report by Kerr, T. H., *Some Geolocation/Geopositioning Considerations for SYERS-2C PIP*, 7 March 2012. **NAVIGATION via Visual Cues Using Only Imaging Sensors:** (2) Rodriguez, J. J., Aggarwal, J. K., "Matching Aerial Images to 3D Terrain Maps," IEEE Trans. on Pattern Analysis and Machine Intelligence,

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Vol. 12, No. 12, pp. 1138-1149, Dec. 1990: Sparse terrain profile data are stored onboard and direct measurement of relative shifts between images are used to estimate position and velocity; however, an EKF is deemed superior here to use of merely a Kalman filter that uses altitude estimates in order to estimate aircraft position and velocity. (3) Heeger, D. J., Jepson, A. D., "Subspace Methods for Recovering Rigid Motion I: Algorithm and Implementation," International Journal of Computer Vision, Vol. 7, No. 2, pp. 95-117, Jan. 1992: Terrain matching methods are also used to estimate platform position and orientation via comparisons to an on-board digital elevation map. (4) Soatto, S., Frezza, R., Perona, P., "Motion Estimation via Dynamic Vision," IEEE Trans. on Automatic Control, Vol. 41, No. 3, pp. 95-117, Mar. 1996: A least squares formulation is used to recover user's 3D motion (3 translation variables and 6 rotation variables or 4 if quaternions are utilized). (5) Goyurfil, P., Rotstein, H., "Partial Aircraft State Estimation from Visual Motion Using the Substate Constraint Approach," AIAA Journal of Guidance, Control, and Dynamics, Vol. 24, No. 5, pp. 1016-1025, Sep.-Oct. 2001: What is called an implicit EKF is used here to estimate aircraft states-aircraft velocities, angular rates, angle of attack, and angle of sideslip but not aircraft Euler angles nor inertial location. Measurements available are the image points of N featured objects, which are tracked from one frame to another. (6) Hoshizaki, T., Andrisani, D., Braun, A. W., Mulyana, A. K., and Bethel, J. S., "Performance of Integrated Electro-Optical Navigation Systems," Navigation: Journal of the Institute of Navigation, Vol. 51, No. 2, pp. 101-122, Summer 2004: Contains good modeling and they have a "tightly coupled system consisting of INS, GPS, and EO" all working together to simultaneously benefit both navigation and photogrammetry (estimates platform states, sensor biases, and unknown ground object coordinates using a single Kalman filter). **Use of control points avoided pre-stored terrain.** (7) Kyungsuk Lee, Jason M. Kriesel, Nahum Gat, "Autonomous Airborne Video-Aided Navigation," Navigation: Journal of the Institute of Navigation, Vol. 57, No. 3, pp. 163-173, Fall 2010: ONR-funded discussion utilizes (1) "digitally stored georeferenced landmark images" (altimeter/DTED), (2) video from an onboard camera, and (3) data from an IMU. Relative position and motion are tracked by comparing simple mathematical representations of consecutive video frames. A single image frame is periodically compared to a landmark image to determine absolute position and to correct for possible drift or bias in calculating the relative motion. (8) Craig Lawson, John F. Raquet, Michael J. Veth, "The Impact of Attitude on Image-Based Integrity," Navigation: Journal of the Institute of Navigation, Vol. 57, No. 4, pp. 249-292, Winter 2010: Being aware of the historical importance of having good satellite geometry when seeking to utilize GPS for positioning and for timing (characterized by HDOP, VDOP, TDOP, and GDOP), they analogously extrapolate these ideas to the geometry of their airborne image collecting and refer to this as image integrity (similar to how researchers endeavor to associate sufficient Integrity to GPS measurements). Known a/c attitude significantly beats unknown attitude (altitude-indexed). **All of the above likely comparable to Classified Pointing Improvements:** Cobra Ball/Cobra Eye & airborne Laser developments!

19 Mar 2018

Some technical notes on the subject of operations counts of SVD or Eigenvalue-Eigenvector decomposition and other approaches to establishing Positive definiteness/semidefiniteness of symmetric matrices (as routinely arise in implementing Kalman filters): (1) Kerr, T. H., "Testing Matrices for Definiteness and Application Examples that Spawn the Need," AIAA Journal of Guidance, Control, and Dynamics, Vol. 10, No. 5, pp. 503-506, Sept.-Oct., 1987. (2) Kerr, T. H., "On Misstatements of the Test for Positive Semidefinite Matrices," AIAA Journal of Guidance, Control, and Dynamics, Vol. 13, No. 3, pp. 571-572, May-Jun. 1990. (as occurred in Navigation & Target Tracking software in the 1970's & 1980's using

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counterexamples) **(3)** Kerr, T. H., "Fallacies in Computational Testing of Matrix Positive Definiteness/Semidefiniteness," IEEE Transactions on Aerospace and Electronic Systems, Vol. 26, No. 2, pp. 415-421, Mar. 1990. [Lists fallacious algorithms that the author found to routinely exist within U.S. Navy submarine navigation and sonobuoy software in the late 1970's and early 1980's using counterexamples to point out the problems.] The third publication above has the greatest relevance in its last Section VII so readers can recognize the problems if/when they ever see them in software again **(which is typically the case)**. The specific applications exhibiting the particular problems discussed in the open literature are only explicitly identified in the corresponding **Intermetrics IV&V reports** (delivered only to the pertinent NAVY customers).

My endorsement for use of Beirman's and Thornton's **U-D-U^T** squareroot Kalman filter formulation in Section VII was OK for its time (1990 and before). **However, by the late 1990's, computers were built differently and computation of the scalar square root was no longer iterative and so no longer as time consuming. Consequently, Neal Carlson's squareroot filter has the least computations or operations counts even though it uses explicit computation of scalar square roots (now, more easily handled in hardware, using logarithm and anti-logarithm implementation).**

22 Mar 2018

Certain errors previously existed within the methodology for how various aerospace companies computed the matrix exponential (as used for calculating the transition matrix for Linear Time-Invariant [LTI] systems), as associated with use of mathematical norms: (1) Kerr, T. H., "A Simplified Approach to Obtaining the Steady-State Initial Conditions for Linear System Simulations," Proceedings of the Fifth Annual Pittsburgh Conference on Modeling and Simulation, 1974. **(2)** Kerr, T. H., "An Invalid Norm Appearing in Control and Estimation," IEEE Transactions on Automatic Control, Vol. 23, No. 1, February 1978. **(3)** Kerr, T. H., "Three Important Matrix Inequalities Currently Impacting Control and Estimation Applications," IEEE Transactions on Automatic Control, Vol. 23, No. 6, December 1978. **(4)** Kerr, T. H., "The Principal Minor Test for Semidefinite Matrices-Author's Reply," AIAA Journal of Guidance, Control, and Dynamics, Vol. 13, No. 3, p. 767, Sep.-Oct. 1989. **(5)** Kerr, T. H., "The Proper Computation of the Matrix Pseudo-Inverse and Its Impact in MVRO Filtering," IEEE Transactions on Aerospace and Electronic Systems, Vol. 21, No. 5, September 1985. **(6)** Kerr, T. H., and Satz, H., S., "Applications of Some Explicit Formulas for the Matrix Exponential in Linear Systems Software Validation," Proceedings of 16th Digital Avionics System Conference, Vol. I, pp. 1.4-9 to 1.4-20, Irvine, CA, 26-30 Oct. 1997. The situation is much better today thanks to the eternal vigilance of numerical analysts and application engineers (such as those at The MathWorks, who recognize its importance)! **For a quick summary of standard approximations invoked and utilized within many Kalman filter implementations and to provide some closed-form analytic solutions that can be useful in verifying/validating the correctness of a Kalman filter software implementation, please see: (7)** Kerr, T. H., "Numerical Approximations and Other Structural Issues in Practical Implementations of Kalman Filtering," a chapter in Approximate Kalman Filtering, edited by Guanrong Chen, World Scientific, NY, 1993. **(8)** Kerr, T. H., "Exact Methodology for Testing Linear System Software Using Idempotent Matrices and Other Closed-Form Analytic Results," Proceedings of SPIE, Session 4473: Tracking Small Targets, pp. 142-168, San Diego, 29 July-3 Aug. 2001. **(closed-form analytic solutions)**

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Besides merely limiting one's attention to the Kalman filter algorithm itself and its mechanization in software, one must occasionally branch out and fix glitches made by others in abutting technology areas, which can occur in models to be used: fairly recently incurred by someone attempting to handle numerical gravity data using what was claimed to be a new technique for system realization (that was not their area of specialization nor area of familiarity), as corrected in: **(1)** Kerr, T. H., "Comment on 'Precision Free-Inertial Navigation with Gravity Compensation by an Onboard Gradiometer'," AIAA Journal of Guidance, Control, and Dynamics, Vol. 30, No. 4, pp. 1214-1215, Jul.-Aug. 2007. (Specifically, two counterexamples are provided where previously claimed conditions are demonstrated to not be satisfied as claimed to be the case.) Another is in simplifying a test proposed by others for two ellipsoid overlap, when the ellipsoids are not necessarily related (as they are in the CR2 failure detection approach above in Item 5 of 16), as availed in: **(2)** Kerr, T. H., "Comments on 'Determining if Two Solid Ellipsoids Intersect'," AIAA Journal of Guidance, Control, and Dynamics, Vol. 28, No. 1, pp. 189-190, Jan.-Feb. 2005. By simplifying a GPS-related optimization problem by revealing that it has a directly calculated closed-form solution as: **(3)** Kerr, T. H., "Comment on 'Low-Noise Linear Combination of Triple-Frequency Carrier Phase Measurements'," Navigation: Journal of the Institute of Navigation, Vol. 57, No. 2, pp. 161, 162, Summer 2010. Frequently having to deal with principles of operation of the actual hardware and identifying likely vulnerabilities way before others saw them and started fixing them: **(4)** Kerr, T. H., "Further Critical Perspectives on Certain Aspects of GPS Development and Use," Proceedings of 57th Annual Meeting of the Institute of Navigation, pp. 592-608, Albuquerque, NM, 9-13 Jun. 2001. (An expose of several loose ends in GPS development that needed [and have now received] further attention before unabated and unabashed reliance upon GPS, as had been claimed to be the plan in the late 1990's for Battlefield 2000.)

1 Apr 2018

Decentralized Kalman filter stability for navigation, as investigated for the U.S. Navy Joint Tactical Information Distribution Systems (JTIDS)-RelNav: **1.** Kerr, T. H., "Stability Conditions for the RelNav Community as a Decentralized Estimator-Final Report," Intermetrics, Inc. Report No. IR-480, Cambridge, MA, 10 Aug. 1980, for NADC (Warminster, PA). **2.** Kerr, T. H., and Chin, L., "A Stable Decentralized Filtering Implementation for JTIDS RelNav," Proceedings of IEEE Position, Location, and Navigation Symposium (PLANS), Atlantic City, NJ, 8-11 Dec. 1980. **3.** Kerr, T.H., and Chin, L., "The Theory and Techniques of Discrete-Time Decentralized Filters," in Advances in the Techniques and Technology in the Application of Nonlinear Filters and Kalman Filters, edited by C.T. Leondes, NATO Advisory Group for Aerospace Research and Development, AGARDograph No. 256, Noordhoff International Publishing, Lieden, pp. 3-1 to 3-39, 1981. **4.** Kerr, T. H., "Extending Decentralized Kalman Filtering (KF) to 2D for Real-Time Multisensor Image Fusion and/or Restoration: Optimality of Some Decentralized KF Architectures," Proceedings of the International Conference on Signal Processing Applications & Technology (ICSPAT96), Boston, MA, 7-10 Oct. 1996. **For the U.S. Navy:** **5.** Kerr, T. H., "Impact of Navigation Accuracy in Optimized Straight-Line Surveillance/Detection of Undersea Buried Pipe Valves," Proceedings of National Marine Meeting of the Institute of Navigation (ION), Cambridge, MA, 27-29 Oct. 1982. For GPS (a representative partial listing): **6.** Kerr, T. H., "Phase III GPS Integration; Volume 1: GPS U.E. Characteristics," Intermetrics Report IR-MA-177, Cambridge, MA, Jan. 1983, for Navair. **7.** Kerr, T.H., "GPS/SSN Antenna Detectability," Intermetrics Report No. IR-MA-199, Cambridge, MA, 15 Mar. 1983, for NADC For U.S. Navy minesweeper navigation: **8.** Kerr, T. H., and Rogers, R., "Report on PINS Filter Design Review (of Magnavox)," Intermetrics Memo, Cambridge, MA, 11 Aug. 1983, for NOSC (San Diego, CA). **For DARPA:** **9.** Kerr, T. H., "Use of GPS/INS in the Design of Airborne Multisensor Data Collection Missions (for Tuning NN-based ATR algorithms)," the Institute of Navigation Proceedings of GPS-94, Salt Lake City, UT, pp. 1173-1188, 20-

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23 Sep. 1994. **10.** Kerr, T. H., "A Critique of Neural Networks as Currently Exist for Control and Estimation," Proc. of the International Conference on Signal Processing Applications & Technology (ICSPAT), Boston, MA, pp. 1434-1443, 24-26 Oct. 1995. **11.** Kerr, T. H., "Critique of Some Neural Network Architectures and Claims for Control and Estimation," IEEE Transactions on Aerospace and Electronic Systems, Vol. 34, No. 2, pp. 406-419, Apr. 1998. *(Extends beyond prior version in item 10, above.)

1 Apr 2018

For the U.S. Air Force: **1.** Carlson, N. A., Kerr, T. H., Sacks, J. E., "Integrated Navigation Concept Study," Intermetrics Report No. IR-MA-321, 15 Jun. 1984, for ITT (Nutley, NJ). **2.** Kerr, T. H., "Decentralized Filtering and Redundancy Management for Multisensor Navigation," IEEE Trans. on Aerospace and Electronic Systems, Vol. 23, No. 1, pp. 83-119, Jan. 1987 (minor corrections appear on p. 412 of May and on p. 599 of Jul. 1987 issues of same journal). **3.** Kerr, T. H., "Comments on 'Federated Square Root Filter for Decentralized Parallel Processes'," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-27, No. 6, Nov. 1991. **For U.S. Navy sonar and sonobuoy tracking filters:** **4.** Kerr, T. H., "Functional and Mathematical Structural Analysis of the Passive Tracking Algorithm (PTA)," Intermetrics Report No. IR-MA-208, Cambridge, MA, 25 May 1983, for NADC. **5.** Kerr, T. H., "Assessment of the Status of the Current Post-Coherent Localization Algorithm," Intermetrics Report No. IR-MA-319, 31 May 1984, for NADC. **For U.S. helicopter missile warning system (MWS):** **6.** Kerr, T. H., "Update to and Refinement of Aspects of Pattern Recognition Principles Used in the Missile Warning System (AN/AAR-47)," Intermetrics Report No. IR-MA-362, 15 Sep. 1983, for Honeywell Electro-Optical.

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An open question (existing for numerical analysts either at The MathWorks or elsewhere) that still needs a definitive answer pertains to the possible universal validity of an alternate approach to direct calculation of the pseudo-inverse matrix that is NOT based on invoking the Singular Value decomposition (SVD). A flow diagram of this alternate approach appears in: **(2)** Kalman, R. E., Englar, T. S., "A User's Manual for the Automatic Synthesis Program (Program C)," NASA Contract Rep. NASA CR-475, June, 1966. A French researcher vouches for its validity in: Proceedings of the International Conference on Signal Processing Applications & Technology, Boston, MA, 7-10 Oct. 1996. The only limitation identified by this French researcher was that this alternative algorithm is not amenable to parallel implementations as a Systolic Array nor as a Cordic Algorithm; while SVD-based Pseudo-inverse calculation is amenable to such. The algorithm discussed immediately above and said to be by French researchers appeared in **(3)** O. Caspary and P. Nus, "Implementation of the Greville algorithm on a Motorola DSP96002 Application to Least-Squares problems," The Proceedings of the 7th International Conference on Signal Processing Applications & Technology (ICSPAT), Boston, MA, USA, pp. 142-145, 7-

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Introduction: Time synchronization is an important issue in multihop ad hoc wireless networks such as sensor networks. Many applications of sensor networks need local clocks of sensor nodes to be synchronized, requiring various degrees of precision. Some intrinsic properties of sensor networks, such as limited resources of energy, storage, computation, and bandwidth, combined with potentially high density of nodes can make traditional synchronization methods unsuitable for these networks; hence, there has been an increasing research focus on designing synchronization algorithms specifically for sensor networks. This article reviews the time synchronization problem and the need for synchronization in sensor networks, then presents, in detail, the basic synchronization methods explicitly designed and proposed for sensor networks.

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