

Completely Sanitized

Thomas H. Kerr III

TeK Associates

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Weighted Least Squares (WLS) Estimation Option:

When measurements are available en masse as:

$$\underline{\mathbf{Z}}^T = [z_1, z_2, \dots, z_N] \quad \underline{\mathbf{H}}^T = [\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_N]$$

as represented using corresponding super observation matrix:
and (with additive white zero mean measurement noise present) as:

$$z_i = \mathbf{H}_i \mathbf{x} + \mathbf{v}_i \quad \Rightarrow \quad \underline{\mathbf{Z}} = \underline{\mathbf{H}} \mathbf{x} + \underline{\mathbf{V}}$$

and the associated measurement noise Covariance matrix in block diagonal form is:

$$\underline{\mathbf{R}} \equiv E[\underline{\mathbf{V}} \underline{\mathbf{V}}^T] = \text{diag}[\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_N]$$

The Weighted Least Squares estimator that minimizes the appropriately weighted cost function:

$$C(\mathbf{x}) = \frac{1}{2} [\underline{\mathbf{Z}} - \underline{\mathbf{H}} \mathbf{x}]^T \underline{\mathbf{R}}^{-1} [\underline{\mathbf{Z}} - \underline{\mathbf{H}} \mathbf{x}] \quad \Rightarrow \quad \mathbf{0}_N = \frac{\partial C(\mathbf{x})}{\partial \mathbf{x}} = -[\underline{\mathbf{Z}} - \underline{\mathbf{H}} \mathbf{x}]^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}}$$

is of the familiar form:

$$\hat{\mathbf{x}}_{\text{WLS}} = [\underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}}]^{-1} \underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{Z}} \quad \text{or} \quad \hat{\mathbf{x}}_{\text{WLS}} = \left[\sum_{i=1}^N \mathbf{H}_i^T \mathbf{R}_i^{-1} \mathbf{H}_i \right]^{-1} \left[\sum_{i=1}^N \mathbf{H}_i^T \mathbf{R}_i^{-1} z_i \right]$$

- WLS needs a fairly large $n \times n$ matrix inversion each time we want the estimate computed.
- However, a recursive EKF estimator represents a **lesser computational burden** and the required **inverse is of a much lower $m \times m$ dimension, where H is $m \times n$ and (usually) $m < n$.**

Weighted Least Squares (WLS) Estimation Option (Cont.'d) :

- Effective covariance of WLS estimation error:

$$\hat{\mathbf{x}}_{\text{WLS}} = \left[\underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \right]^{-1} \underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{z}}$$

so now post-multiplying by its transpose and

taking expectations throughout (as demonstrated below) yields :

$$\mathbf{E} \left[(\hat{\mathbf{x}}_{\text{WLS}} - \mathbf{E}[\hat{\mathbf{x}}_{\text{WLS}}]) (\hat{\mathbf{x}}_{\text{WLS}} - \mathbf{E}[\hat{\mathbf{x}}_{\text{WLS}}])^T \right] = \left[\underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \right]^{-1} \underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \mathbf{E}[\underline{\mathbf{v}} \underline{\mathbf{v}}^T] \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \left[\underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \right]^{-1}$$

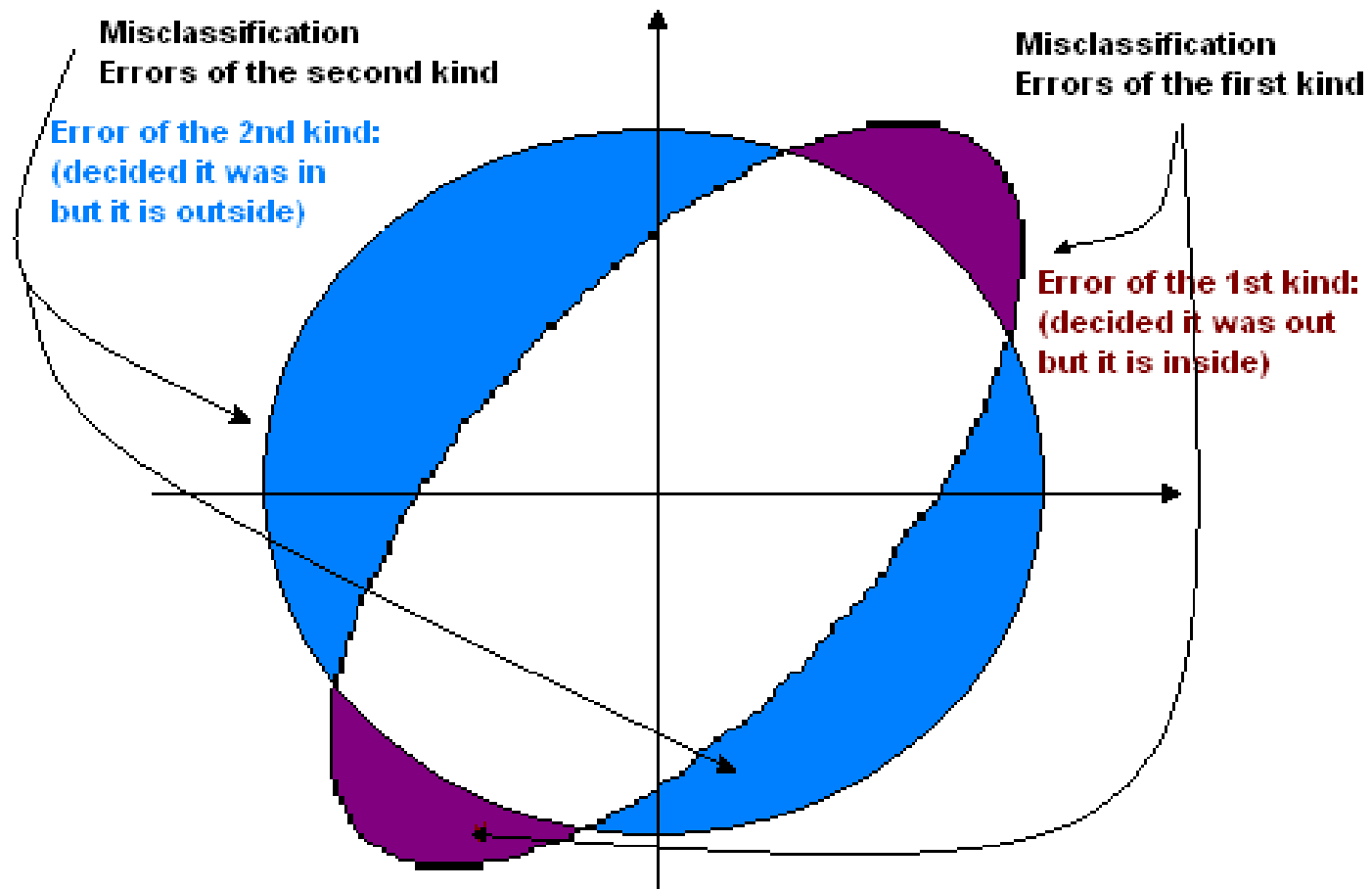
$$\mathbf{E} \left[(\hat{\mathbf{x}}_{\text{WLS}} - \mathbf{E}[\hat{\mathbf{x}}_{\text{WLS}}]) (\hat{\mathbf{x}}_{\text{WLS}} - \mathbf{E}[\hat{\mathbf{x}}_{\text{WLS}}])^T \right] = \left[\underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \right]^{-1} \underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{R}} \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \left[\underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \right]^{-1}$$

$$\mathbf{E} \left[(\hat{\mathbf{x}}_{\text{WLS}} - \mathbf{E}[\hat{\mathbf{x}}_{\text{WLS}}]) (\hat{\mathbf{x}}_{\text{WLS}} - \mathbf{E}[\hat{\mathbf{x}}_{\text{WLS}}])^T \right] = \left[\underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \right]^{-1} \underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \left[\underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \right]^{-1} \text{ thus simplifying to be :}$$

$$\mathbf{E} \left[(\hat{\mathbf{x}}_{\text{WLS}} - \mathbf{E}[\hat{\mathbf{x}}_{\text{WLS}}]) (\hat{\mathbf{x}}_{\text{WLS}} - \mathbf{E}[\hat{\mathbf{x}}_{\text{WLS}}])^T \right] = \left[\underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \right]^{-1}$$

$$\mathbf{P} \equiv \mathbf{E} \left[(\hat{\mathbf{x}}_{\text{WLS}} - \mathbf{E}[\hat{\mathbf{x}}_{\text{WLS}}]) (\hat{\mathbf{x}}_{\text{WLS}} - \mathbf{E}[\hat{\mathbf{x}}_{\text{WLS}}])^T \right] = \left[\underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}} \right]^{-1} = \left[\sum_{i=1}^N \mathbf{H}_i^T \mathbf{R}_i^{-1} \mathbf{H}_i \right]^{-1}$$

Decisions made using ellipsoidal covariances associated with underlying Gaussian processes (in incorrectly classifying points that fall within the two regions below that exhibit descriptive colors)



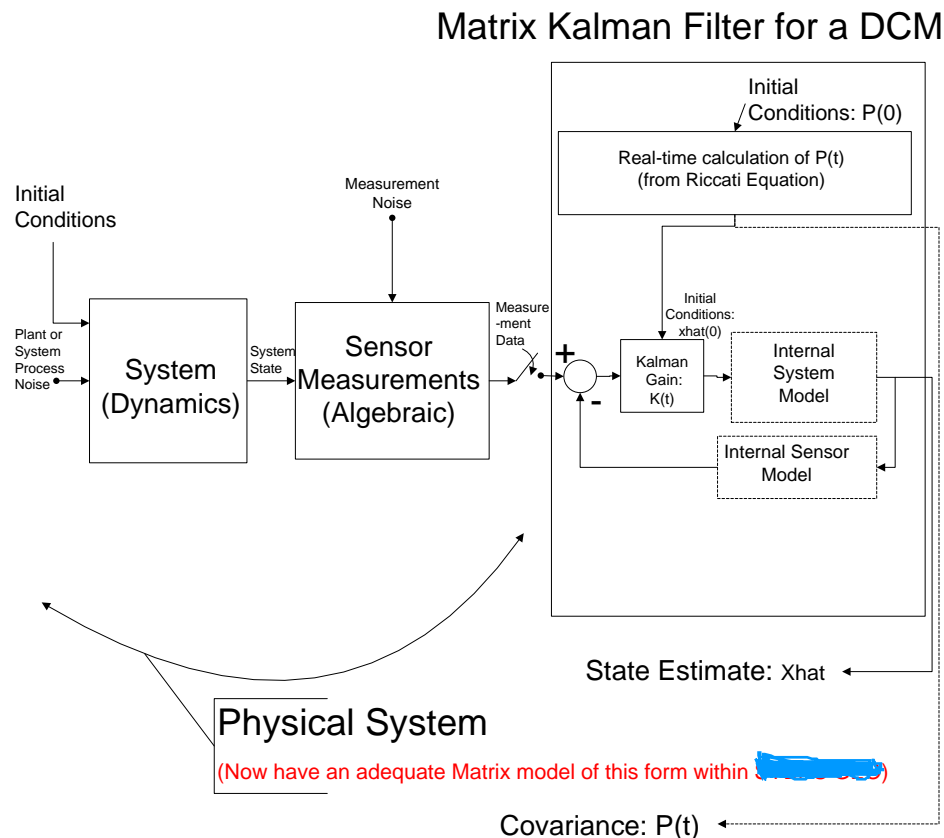
What this means in 2D regarding possibility for wrong conclusions when ellipsoid "tilt" is ignored

(tilted ellipsoid is truth but untilted ellipsoid is what is used in making inclusion decisions)

New Geolocation Improvement Approach

MKF asynchronously updated as measurements become available

Needs **two separate simultaneous sensor measurements** of landmarks in order to infer angle



Physical System

(Now have an adequate Matrix model of this form within [redacted])

Once completed, quick comparisons can be made as $|P(t)| \geq 10^N$ (for critically specified N) to either rely on or ignore whole approach

The above diagram is same as structural form needed to implement Matrix Kalman Filter for DCM!

New Geolocation Improvement Approach:
MKF asynchronously updated each time new measurement becomes available
& Details of Testing Covariance Magnitude

(**Obvious cross-check:** for $j = k-1$ or $t_j = t_{k-1}$, this asynchronous form agrees exactly with synchronous periodic form.)
 For distinctly asynchronous time points $t_k > t_j$ **not** necessarily related by a consistently constant time step Δ :

	PROPAGATE STEP	UPDATE STEP
FILTER COVARIANCE	$\mathbf{P}_{k j} = \Phi(k,j)\mathbf{P}_{j j}\Phi^T(k,j) + \mathbf{Q}_k$	$\mathbf{P}_{k k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k j}(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T$
KALMAN GAIN	$\mathbf{K}_k = \mathbf{P}_{k j}\mathbf{H}_k^T \left[\mathbf{H}_k\mathbf{P}_{k j}\mathbf{H}_k^T + \mathbf{R}_k \right]^{-1}$	
FILTER ESTIMATE	$\hat{\mathbf{X}}_{k j} = \Phi(t_k, t_j)\hat{\mathbf{X}}_{j j}$	$\hat{\mathbf{X}}_{k k} = \hat{\mathbf{X}}_{k j} + \mathbf{K}_k \left[\mathbf{z}_k - \mathbf{h}(\hat{\mathbf{X}}_{k j}) \right]$

Transition matrix $\Phi(\cdot, \cdot)$ in the above is obtained from the Jacobian of the system matrix, $f(\cdot)$, evaluated at $\hat{x}(t_j|t_j)$ and H_j is the Jacobian of the observation matrix, $h(\cdot)$, evaluated at $\hat{x}(t_j|t_j)$. When time steps are large, recommend using iterated EKF (with 3 fixed iterations) to improve H_k .

Criterion for checking $|\mathbf{P}(t)| \geq 10^N$ can be implemented as $\text{trace}[\mathbf{P}(t)] \geq 10^N$ (**easier**)

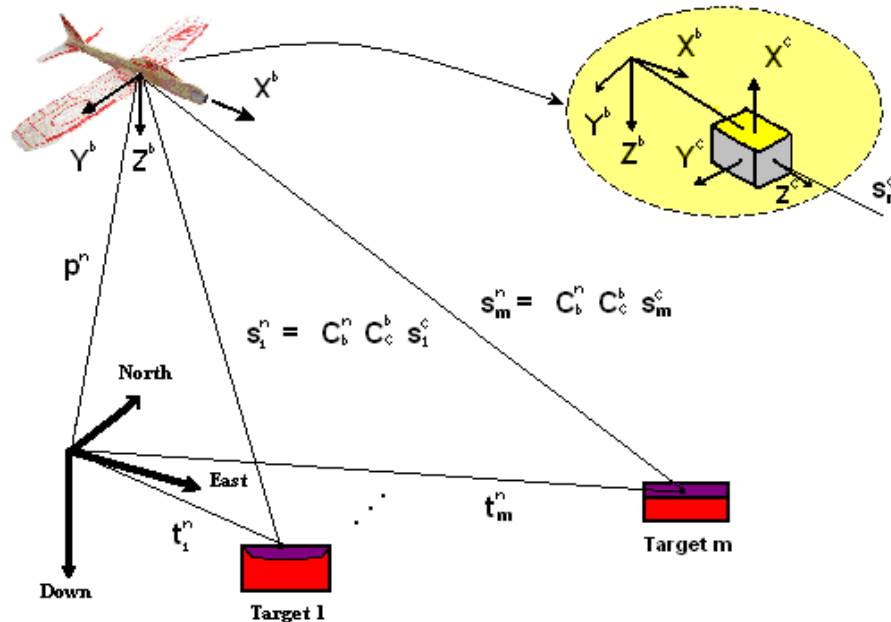
or as

$$\left[\mathbf{P}(t) - 10^N \mathbf{I}_n \right] \geq \mathbf{0}_n \quad (\text{use a matrix positive semi-definite test})$$

The above diagram has the same basic structural form needed to implement a **Matrix Kalman Filter (MKF)** for DCM but **computational burden incurred is slightly higher!**

Need Detailed Accounting for LOS Pointing Errors!

Needs **two separate simultaneous sensor measurements** of landmarks in order to infer angle. However, may **either** need to use **2 different independent synchronized sensors** or a way to **extract angle information from optical image from only one primary sensor.**



- Both Master INS and Secondary IMU (LN-200) are implemented in strapdown & transfer aligned as a/c proceeds on mission. Secondary IMU directly on inner gimbal improves visibility into LOS pointing error (previously degraded due to effect of non-rigid flexure between Primary Aircraft INS and camera that missed being accounted for due to presence/compliance of shock isolation system).
- From White Paper research: **We can tie into a new alternate approach to “observability” based on something like geometry of a standard “GPS integrity approach” (very similar to familiar, older GPS-affiliated GDOP geometry) now better matched to imaging targets via a time-to-target window.**

Matching Up with Image Integrity approach to “Observability”

- In the next few slides, we further investigate the following **BLANK** information (conveyed from PDR) in seeking to better match-up with “Image Integrity” approach conveyed in 2010 Institute of Navigation (ION) Paper by Veth et al (see next to last slide herein for a short summary)¹.

¹ Craig Lawson, John F. Raquet, Michael J. Veth, “The Impact of Attitude on Image-Based Integrity,” *Navigation: Journal of the Institute of Navigation*, Vol. 57, No. 4, pp. 249-292, Winter 2010. A summarizing discussion is provided in Appendix A conveying more details (pp. 15-21) of the report by Kerr, T. H., Some Geolocation/Geopositioning Considerations for **BLANK**, 7 March 2012 (found in the Systems Engineering section of the **BLANK** Portal).

AFIT’s Prof. John Raquet (Lt. Col.) became an Institute of Navigation (ION) Fellow recently (at Jan./Feb. 2012 meeting) and became the first ever AFIT Fulbright Scholar.

NAVIGATION via Visual Cues Using Only Imaging Sensors:

- Rodriguez, J. J., Aggarwal, J. K., "Matching Aerial Images to 3D Terrain Maps," *IEEE Trans. on Pattern Analysis and Machine Intelligence*, Vol. 12, No. 12, pp. 1138-1149, Dec. 1990: Sparse terrain profile data are stored onboard and direct measurement of relative shifts between images are used to estimate position and velocity; however, an EKF is deemed superior herein by them to use of merely a Kalman filter that uses altitude estimates in order to estimate aircraft position and velocity.
- Heeger, D. J., Jepson, A. D., "Subspace Methods for Recovering Rigid Motion I: Algorithm and Implementation," *International Journal of Computer Vision*, Vol. 7, No. 2, pp. 95-117, Jan. 1992: Terrain matching methods are also used to estimate platform position and orientation via comparisons to an on-board digital elevation map.
- Soatto, S., Frezza, R., Perona, P., "Motion Estimation via Dynamic Vision," *IEEE Trans. on Automatic Control*, Vol. 41, No. 3, pp. 95-117, Mar. 1996: A least squares formulation is used to recover user's 3D motion (3 translation variables and 6 rotation variables or 4 if quaternions are utilized).
- Goyurfil, P., Rotstein, H., "Partial Aircraft State Estimation from Visual Motion Using the Substate Constraint Approach," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 5, pp. 1016-1025, Sep.-Oct. 2001: What is called an implicit EKF is used here to estimate aircraft states-aircraft velocities, angular rates, angle of attack, and angle of sideslip but not aircraft Euler angles nor inertial location. Measurements available are the image points of N featured objects, which are tracked from one frame to another.
- Hoshizaki, T., Andrisani, D., Braun, A. W., Mulyana, A. K., and Bethel, J. S., "Performance of Integrated Electro-Optical Navigation Systems," *Navigation: Journal of the Institute of Navigation*, Vol. 51, No. 2, pp. 101-122, Summer 2004: Contains good modeling and they have a "tightly coupled system consisting of INS, GPS, and EO" all working together to simultaneously benefit both navigation and photogrammetry (estimates platform states, sensor biases, and unknown ground object coordinates using a single Kalman filter). Use of control points avoided pre-stored terrain.
- 10⁻³ ◦
▪ Kyungsuk Lee, Jason M. Kriesel, Nahum Gat, "Autonomous Airborne Video-Aided Navigation," *Navigation: Journal of the Institute of Navigation*, Vol. 57, No. 3, pp. 163-173, Fall 2010: ONR-funded discussion utilizes (1) "digitally stored georeferenced landmark images" (altimeter/DTED), (2) video from an onboard camera, and (3) data from an IMU. Relative position and motion are tracked by comparing simple mathematical representations of consecutive video frames. A single image frame is periodically compared to a landmark image to determine absolute position and to correct for possible drift or bias in calculating the relative motion.
- ▪ Craig Lawson, John F. Raquet, Michael J. Veth, "The Impact of Attitude on Image-Based Integrity," *Navigation: Journal of the Institute of Navigation*, Vol. 57, No. 4, pp. 249-292, Winter 2010: Being aware of the historical importance of having good satellite geometry when seeking to utilize GPS for positioning and for timing (characterized by HDOP, VDOP, TDOP, and GDOP), they analogously extrapolate these ideas to the geometry of their airborne image collecting and refer to this as image integrity (similar to how researchers endeavor to associate sufficient integrity to GPS measurements). Known a/c attitude significantly beats unknown attitude (altitude-indexed).
- Likely comparable Classified Pointing Improvements: Cobra Ball/Cobra Eye & airborne Laser developments.

Rigorous updates in airborne estimation for attitude determination:

- **Crassidis, J. L., Markley, F. L., Cheng, Y., “Survey of Nonlinear Attitude Estimation Methods,” *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 1, pp. 12-28, Jan. 2007:** An excellent survey on the subject of *attitude estimation*. It provides insights into what is important in estimation algorithms. It is a more practical and rigorous addendum to their many earlier surveys, concerned with utilizing alternative EKF's or Nonlinear Luenberger Observers (as alternatives to Extended Kalman filter-based approaches). They admonish to “stick with EKF”.
- **Majji, M., Junkins, J. L., Turner, J. D., “Jth Moment Extended Kalman Filtering for Estimation of Nonlinear Dynamic Systems,” *AIAA Guidance, Navigation, and Control Conference and Exhibit*, Honolulu, HI, Paper No. AIAA 2008-7386, pp. 1-18, 18-21 Aug. 2008:** Explores two variations on JMEKF formulations that properly handle higher order moments (that lurk in the background while trying to get good estimates and covariances from EKF's). Approximations utilized are acknowledged and properly handled (rather than ignored, as is usually the case). Errors reduced by several orders of magnitude within 5 sec., but results in normalized units (for comparisons to ordinary EKF approach, which it beat by a wide margin). Down side is its larger CPU burden yet to be completely quantified.
- **Scorse, W. T., Crassidis, A. L., “Robust Longitudinal and transverse Rate Gyro Bias Estimation for Precise Pitch and Roll Attitude Estimation in Highly Dynamic Operating Environments Utilizing a Two Dimensional Accelerometer Array,” *AIAA Atmospheric Flight Mechanics Conference*, Paper No. AIAA 2011-6447, Portland, OR, pp. 1-28, 8-11 Aug. 2011:** Using the latest in rigorous real-time estimation algorithms (neither a particle filter nor an unscented/Oxford /Sigma-Point filter) for enabling accurate pointing (precise pitch and roll) within an aircraft within a high dynamics operating environment is reported. While it does utilize rate integrating gyros, as does *BLANK*, it also utilizes 2D accelerometer arrays and compares to an onboard gravity map to achieve its accuracy. Following reasonably large offsets, got back to within 0.1 degree pointing error within 10 seconds but results much worse with turbulence present.
- **Jensen, Kenneth J., “Generalized Nonlinear Complementary Attitude Filter,” *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 5, pp. 1588-1593 , Sept.-Oct. 2011:** Achieves a big breakthrough by providing a proof of this particular EKF's global stability as a consequence by stating that it possesses “almost” *global asymptotic stability*; however, the term “almost” is required terminology to keep probability theorists and purists happy with the wording of his claim. Author Jensen attains his results by utilizing appropriate stochastic Lyapunov functions (proper handling of such due to Prof. Emeritus Harold J. Kushner, Brown Univ.).
- **La Scala, B. F., Bitmead, R. R., James, M. R., “Conditions for stability of the Extended Kalman Filter and their application to the frequency tracking problem,” *Math. Control, Signals Syst. (MCSS)*, vol. 8, No. 1, pp. 1-26, Mar. 1995:** Proof of Stability for yet another EKF. Now worries about EFK divergence evaporate for this application.
- **Reif, K., Gunther, S., Yaz, E., Unbehauen, R., “Stochastic stability of the continuous-time extended Kalman filter,” *Proc. Inst. Elect. Eng.*, Vol. 147, p. 45, 2000:** Proof of Stability for yet another EKF. Now worries about EFK divergence evaporate for this application.
- **Salcudean, S., “A globally convergent angular velocity observer for rigid body motion,” *IEEE Trans. on Autom. Control*, Vol. 36, No. 12, pp.1493-1497, Dec. 1991:** Proof of Stability for alternative Luenberger Observer use too (~EKF).

Rigorous MKF updates in airborne estimation for attitude determination (Cont.'d):

- Choukroun, D., Weiss, H., Bar-Itzhack, I. Y., Oshman, “Kalman Filtering for Matrix Estimation,” *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 42, No. 1, pp. 147-159, Jan. 2006: A linear Matrix Kalman filter for DCM. DCM (Refinement #1)
- Choukroun, D., Weiss, H., Bar-Itzhack, I. Y., Oshman, “Direction Cosine Matrix Estimation from Vector Observations Using a Matrix Kalman Filter,” *AIAA Guidance, Navigation, and Control Conference and Exhibit*, pp. 1-11, Aug. 2003: A linear Matrix Kalman Filter for DMC using either vector or matrix measurement updates. **DCM** Refinement #2 .
- Choukroun, D., “A Novel Quaternion Kalman Filter using GPS Measurements,” *Proceedings of ION GPS*, Portland, OR, pp. 1117-1128, 24-27 Sep. 2002: An alternative viewpoint. (Quaternion Refinement #1.)
- Choukroun, D., Weiss, H., Bar-Itzhack, I. Y., Oshman, “Kalman Filtering for Matrix Estimation,” *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 42, No. 1, pp. 147-159, Jan. 2006: Quaternion Refinement #2.
- Choukroun, D., Bar-Itzhack, I. Y., Oshman, “Novel Quaternion Kalman Filter,” *IEEE Trans. on Aerospace and Electronic Systems*, Vol. 42, No. 1, pp. 174-190, Jan. 2006: Quaternion Refinement #3.
- Choukroun, D., Weiss, H., Bar-Itzhack, I. Y., Oshman, “Direction Cosine Matrix Estimation From Vector Observations Using A Matrix Kalman Filter,” *Proceedings of AIAA Guidance, Navigation, and Control Conference and Exhibit*, Austin, TX, pp. 1-11, 11-14 August 2003: **DCM** Refinement #3
- Choukroun, D., “Ito Stochastic Modeling for Attitude Quaternion Filtering,” *Proceedings of Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, Shanghai, P. R. China, pp. 733-738, 16-18 Dec. 2009: Quaternion Refinement #4.

Matrix KF material that we also seek to exploit for **BLANK** was primarily by Daniel Choukroun, B. S. (Summa cum Laude), M.S., Ph.D. (1997, 2000, 2003), post-doc (UCLA), currently an Assistant Professor at Delft University of Technology, Netherlands.

NASA updates in Spaceborne estimation for attitude determination:

- Cheng, Y., Landis Markley, F., Crassidis, J. L. Oshman, Y., “Averaging Quaternions,” *Advances in the Astronautical Sciences series*, Vol. 127, American Astronautical Society, AAS paper No. 07-213, 2007:
- Landis Markley, F., “Attitude Filtering on $SO(3)$,” *Advances in the Astronautical Sciences series*, Vol. 122, American Astronautical Society, AAS paper No. 06-460, 2006:
- Cheng, Y., Crassidis, J. L., and Landis Markley, F., “Attitude Estimation for Large Field-of-View Sensors,” *Advances in the Astronautical Sciences series*, Vol. 122, American Astronautical Society, AAS paper No. 06-462, 2006:
- Landis Markley, F., “Attitude Estimation or Quaternion Estimation?,” *Advances in the Astronautical Sciences series*, Vol. 115, American Astronautical Society, AAS paper No. 03-264, 2003: **Critical and thorough Analysis of 3 different EKF’s vs. Technion MKF. However, MKF was improved as a consequence.**
- Reynolds, R., Landis Markley, F., Crassidis, J. L., “Asymptotically Optimal Attitude and Rate Bias Estimation with Guaranteed Convergence,” *Advances in the Astronautical Sciences series*, Vol. 132, American Astronautical Society, AAS paper No. 08-286, 2008:

Estimation Results for Bilinear Systems (to tie into the MKF results):

- Halawani, T. U., Mohler, R. R., and Kolodziej, W. J., “A two-step bilinear filtering algorithm,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. 32, 344-352, 1984: **Summarize!**
- Glielmo, L., Marino, P., Setola, R., Vasca, F., “Parallel Kalman Filter Algorithm for State Estimation in Bilinear Systems,” *Proceedings of the 33rd Conference on Decision and Control*, Lake Buena Vista, FL, pp. 1228-1229, Dec. 1994: **Summarize!**
- Wang, Z., Qiao, H., “Robust Filtering for Bilinear Uncertain Stochastic Discrete-Time Systems,” *IEEE Trans. on Signal Processing*, Vol. 50, No. 3, pp. 560-567, Mar. 2002: **“Robust” approaches usually have sluggish response. Lack of timely results usually only useful for process control applications.**
- Lopes dos Santos, P., Ramos, J. A., Frias, R., “Derivation of a Bilinear Kalman Filter with Autocorrelated Inputs,” *Proceedings of the 46th Conference on Decision and Control*, New Orleans, LA, pp, 6196-6202, 12-14 Dec. 2007: **Structure similar to what Technion MKF exhibits.**