in the same manner as [Sec. 2.3]{Kerr?} by providing collaborative comparison of outputs to verify performance of a general EKF implementation (instantiated with the same test case) if both implementations agree (sufficiently) for this simple test. This proposed manner of use for EKF software verification would be in keeping with the overall software test philosophy espoused as it evolved in {Kerr92}, {Kerr93}, {Kerr49}, and concisely summarized and generalized for nonlinear estimation situations in {KerrtoChen} (and which offers a nice simplification of the software check case of {Ramachandra} in its Appendix A).

We are aware of the following approach that has evolved over the last 30+ years (Nahi [1969], Jaffer and Gupta [1971], Hadidi and Schwartz [1979], Monzingo [1975, 1981], Askar and Derin [1984], and Tugnait and Haddad [1975]) to handle situations where there is data dropout or missing data but we will not dwell on it because it complicates the situation well beyond what is needed. For application scenarios with computational architectures that try to force use of a constant uniform step-size throughout the implementation, the lack of measurement returns at step k can be modeled using a scalar independent multiplicative random variable γ_k that takes two possible values, either 0 or 1, within the standard expression for the received measurement data:

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$$z_k \quad = \quad \gamma_k \, h(x_k) + v_k .$$

In the above expression, the missing measurements correspond to $\gamma_k = 0$ for only noise being received. When $\gamma_k = 1$, the desired signal is present in the measurements. The problem with this is no real structure is available for predicting the behavior of γ_k and the above structure makes even applications possessing linear systems and linear measurements become horribly nonlinear and relatively intractable. An architecture that just processes measurements when the received signal exceeds the mandatory detection threshold avoids these problems and is more straight-forward to implement (by not relying on having a constant step-size).

We couldn't conclude without acknowledging likely parallel implementation of the "Bank-of-Kalman-Filters" approach {Popp} (where each filter has a different underlying system model matched or representing a different hypothesized underlying situation) with global probability assessments of each filter possibly coinciding exactly with the true situation (currently prevailing and from which the only measurements are availed throughout) being automatically calculated on-line as an integral part of this methodology, which is totally rigorous only for linear systems. (As originally conceived in 1965 by Magill, popularized by Demitri Laniotis as "partitioned filters", but only relatively recently pursued for actual use by R. Grover Brown, Peter Maybeck, Yaakov Bar-Shalom, Wang Tang (ARINC), Tom Kerrien (Alphatech) within the last 15 years in IR, GPS, Radar, and multi-target sonar and radar applications with significant extensions being provided in the last six years by Y. Bar-Shalom, H. Blom, and X.-R. Li {Bar-Shalom1}, {Bar-Shalom2}.)

Although we are not actively recommending that anyone pursue this approach[48] at this time, please consider the following possibilities in the nonlinear estimation area of Reentry Vehicle target tracking:

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[48] A detail is that originally Magill (in the mid '60's), Laniotis (in the early '70's), John Deyst (in the early '70's), Charles Brown (TASC in the early '70's) used maximization to pick highest probability and choose just a single estimate as the winner, while present day implementations blend the estimates as weighted by corresponding probabilities. Mike

- use of alternative atmospheric models upon reentry, where arguments arise as to exactly what altitude it kicks in.
- alternative RV masses hypothesized (quantized over the finite possibilities aided by prior intelligence gathering to elucidate candidates).
- quantized on possibilities on spin modulation speed (if any) as elucidated by prior intelligence gathering.
- quantized over likely reentry angles (which affects drag and lift). Different countries use different conventions on reentry angle (but may change at the last minute to reap the element of surprise just like in the Electronic Intelligence [ELINT] game of ongoing Electronic Warfare [EW]).

A "bank-of-Kalman-filters" is also being used in some simplified approximate multitarget tracking methodologies such as the Joint Probabilistic Data Association (JPDA) scheme advertised by Y. Bar-Shalom[49] as being a lesser computational burden than full Multi-Hypothesis Test (MHT).

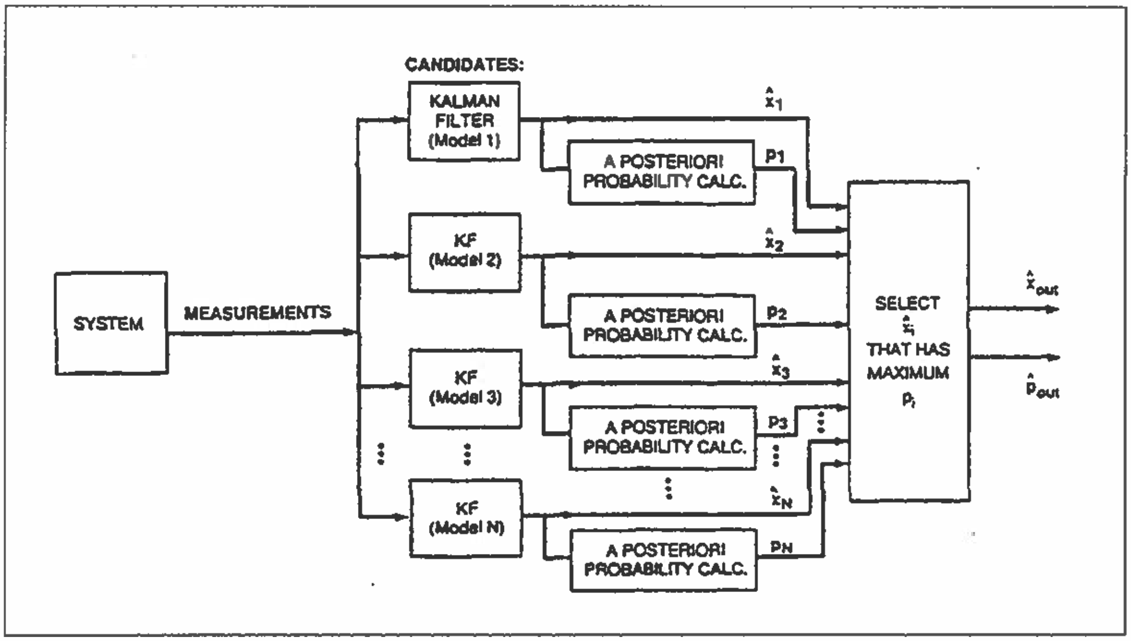

Figure A.1: Multiple Model of Magill (MMM): N alternative filters, each with its distinctly different system models, vying to match the true (unknown) system as it progresses through its likely operating regimes (characterized a priori by analysts), with associated on-line computation of probabilities of each being correct so that a tally is available to decide which one (choice of a "winner" varying with time) offers the best match

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Athans et al was the first to blend outputs like this in Oct. 1977 issue of *IEEE Trans. on Automatic Control* on "Fly-by-Wire Control of the F-8 Test Aircraft" but they acknowledged that doing so was heuristic.

[49] Yaakov Bar-Shalom and Hank Blom also use ageneralization of MMM (denoted as IMM) and have a nice description of the accompanying probability calculations of IMM, which in turn, determines which running filter model most closely corresponds to actual measurements received. Bar-Shalom and X.-R. Li have recently extended this structure to automatically close down on the number of model filters to avoid an excess of candidates (that would otherwise drain computer resources and water down tracking performance as well).

Prof. Nikias' recent textbook {Nikias} offers nice results at the extreme of fractional moment estimation (i.e., in the presence of fat tailed distributions [as occurs when bombarded with statistical outliers or mavericks being prevalent]) which he calls α-stable distributions. We want to eventually include a consideration of this methodology as one of our tasks since we feel that this approach is newly elaborated, rigorous, and has great potential but needs some further development work to bring it to fruition for Kalman filtering for RV tracking although there is already a scalar KF precedent {Stuck} to serve as a lead stepping stone (but more analysis and derivation work needs to be done for the general case).

2.3.A.4 Extending Linear Estimation Techniques to Nonlinear Systems: the linearization}

The underlying mathematical models describing target tracking are usually of the form of a nonlinear ordinary differential equation (ODE):

$$\dot{x}(t) = f(x, u, t) \ , \qquad \text{(A.1)}$$

where *u(t)* is an input driving function (either a deterministically specified control[50] or zero mean white Gaussian process noise or both but can be absent or zero). The measurement equation associated with using a tracking sensor such as radar is of the form of a nonlinear algebraic equation:

$$z(t) = h(x, v, t) \ , \qquad \text{(A.2)}$$

where *v(t)* is zero mean white Gaussian noise, independent of the *u(t)* that appears in Eq.~2.5.

To linearize the general system of Eqs.~2.4 and 2.5 about an operating point $\bar{x}$, i.e., the state space trajectory resulting from applying the control $\bar{u}$ *(t)*, a Taylor series approximation is used and expanded about $\bar{x}$ as

$$\begin{aligned} \dot{\bar{x}} + \partial\dot{x} &= f(\bar{x} + \partial x, \bar{u}(t) + \partial u(t), t) \\ &= f(\bar{x}, \bar{u}, t) + \frac{\partial f}{\partial x}\Big|_{x=\bar{x}} (\bar{x} + \partial x - \bar{x}) + \frac{\partial f}{\partial u}\Big|_{u=\bar{u}} (\bar{u} + \partial u - \bar{u}) + H.O.T. \ , \end{aligned} \qquad \text{(A.3)}$$

and, correspondingly, with

$$\bar{z} + \partial z = h(\bar{x}, \bar{v}, t) + \frac{\partial h}{\partial x}\Big|_{x=\bar{x}} (\bar{x} + \partial x - \bar{x}) + \frac{\partial h}{\partial v}\Big|_{v=\bar{v}} (\bar{v} + \partial v - \bar{v}) + H.O.T. \ , \qquad \text{(A.4)}$$

and,

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(where the vertical bar here is a reminder and denotes evaluation of both Jacobians at $x=\bar{x}$ and $u=\bar{u}$) and the original system equation for the particular trajectory $\bar{x}$:

$$\dot{\bar{x}} = f(\bar{x}, \bar{u}, t) \tag{A.5}$$

is then subtracted en masse from Eq. A.3 to result in

$$\partial \dot{x} = \left[\frac{\partial f}{\partial x}|_{x=\hat{x}}\right] \partial x + \left[\frac{\partial f}{\partial u}|_{u=\bar{u}}\right] \partial u \,, \tag{A.6}$$

and the original measurement equation, evaluated along the trajectory $\bar{x}$:

$$\bar{z}(t) = h(\bar{x}, \bar{v}, t) \,, \tag{A.7}$$

is then subtracted en masse from Eq. A.4 to result in

$$\partial z(t) = \left[\frac{\partial h}{\partial x}|_{x=\hat{x}}\right] \partial x + \left[\frac{\partial h}{\partial v}|_{v=\bar{v}}\right] \partial v \,, \tag{A.8}$$

where the higher order terms (H.O.T.) are ignored and dropped from further consideration because they are small in comparison to primary terms. In keeping with established tradition, instead of writing the exacting expression ∂x for all the deviations in the now linearized versions of Eqs.~2.9 and 2.11, we now just drop the leading symbol ∂ throughout for convenience while acknowledging that it is actually the deviations that we now have in these states of the linearized model (expressed in the same units). This version of a linear error model is now compatible with the form needed for Kalman filtering (corresponding to an *F(t)* and *H(t)*, respectively, in Eqs. 2.12 and 2.13 below). To invoke use of an Extended Kalman Filter for a nonlinear system, the above described linearization should be performed about the best *available* estimate $\bar{x} = \hat{x}_{k|k-1}$ or possibly after an iterative relinearization at each time step as an extended iterated EKF {Kerr?} (see Table~1), or by using more terms from the Taylor series in performing the linearization as a so-designated *Gaussian Filter* or *second-order filter*[51] of slightly different constructions {Jazwinski}, {Liang}, {Satz}, {Widnall}, {Satz2}, {Nam}.

The standard linear dynamical system for which Kalman-type filters are designed has a continuous-time representation consisting of an *n*-dimensional state vector[52] *x(t)* and an *p*-dimensional measurement vector *z(t)* of the form:

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[51] Some confusion exists since {Bar-Shalom} refers to both of these situations as Extended Kalman Filters but most other references distinguish between these two filters by making a distinction in their names.

[52] Representative details of the specific states and measurements and transformations and coordinate conventions used in RV target tracking via radar are conveyed in {Kerr?}.

$$\text{System}: \quad \dot{x}(t) = F(t)\,x(t) + G(t)\,w(t), \tag{A.9}$$

$$\text{Sensor Measurements}: \quad z(t) = H(t)\,x(t) + v(t), \tag{A.10}$$

and a corresponding associated discrete-time representation consisting of an n-dimensional state vector x_k and a p-dimensional measurement vector z_k of the following well-known form:

$$\text{System}: \quad x_{k+1} = \Phi(k+1,k)x_k + w_k, \tag{A.11}$$

$$\text{Sensor Measurements}: \quad z_k = H_k x_k + v_k, \tag{A.12}$$

with random initial condition $x(0)$ from a Gaussian distribution $x(0) \sim \mathcal{N}(\bar{x}_0, P_0)$ (of known mean and variance) and where $\Phi(k+1,k)$ is the known *transition matrix* and the process and measurement noises, w_k and v_k, respectively, are zero mean, white Gaussian noises (independent of the Gaussian initial condition) of known covariance intensity levels[53] Q_k and R, respectively. The two symmetric matrices P_0 and Q_k must be at least positive semi definite and the third symmetric matrix R must usually be positive *definite*. Although considerable historical confusion persists on this topic of how to correctly confirm positive semi-definiteness numerically, proper computational tests of these required properties exist, as discussed in {Kerr43}-{Kerr46}, where this same topic arises again in testing for observability in some formulations of angle-only tracking. The usual regularity conditions of observability/controllability are assumed to be satisfied here by the system of Eqs. A.9, A.10, A.11, and A.12 as has been the case for standard radar applications (but is more challenging to show for angle-only tracking situations and, as such, is still evolving {Taff}, {Jauffret}, {Rao}, {Becker} {Guerci}).

[53] See {Tsang}, {Peters} if they need to be determined.

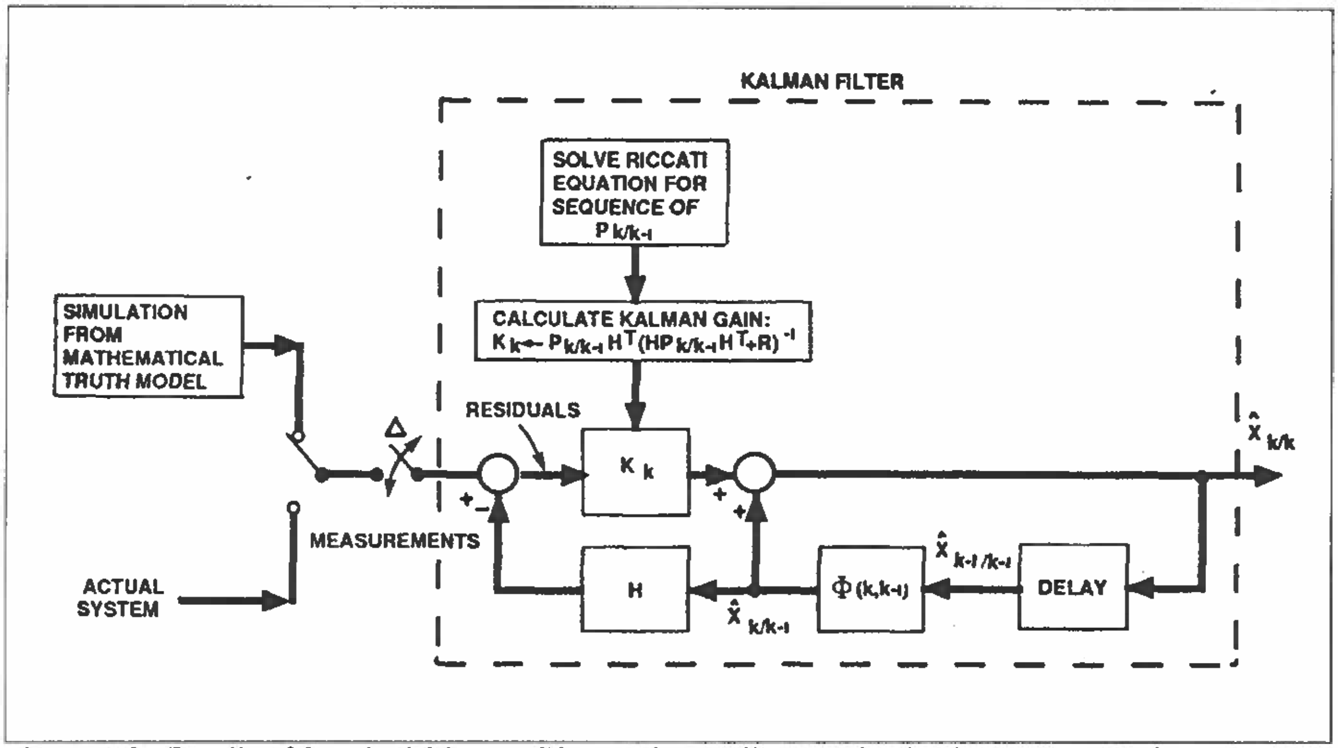

Figure A.2: Details of Standard Discrete-Time Kalman Filter Mechanization as Kernal of an Extended Kalman Filter (applicable to just a simulation or for use within the actual system)

Having the observability/controllability conditions satisfied, as mentioned above, guarantees that the covariance calculations from the associated Riccati equation will be well-behaved and consequently that the resulting KF will be stable in the linear case. The rigorous demonstrations or proofs of these conclusions on the stability exhibited by the KF estimates usually utilizes a Lyapunov function (e.g., {Deyst}, [Sec. 4]{Kerr20}, [Appendix C]{McGarty}), constructed as a quadratic form using the inverse of the covariance, $P^{-1}_{k|k}$, obtained from solution of the associated Riccati equation, and used in the role of an inner product matrix. For nonlinear estimation applications such as RV target tracking, these stability considerations are not exact but merely approximate gauges of utility and should be viewed as suspect since they are frequently entirely useless except for the case of Eqs.~2.4 and 2.5 being purely linear from the start. Similarly, the variance computed via the Riccati equation has little actual significance in the nonlinear estimation scenario of RV tracking,

where ensemble sample averages have more relevance (with issues of sample size and distribution-free tests entering the picture since Gaussianness is absent in nonlinear estimation, as discussed in Sec. 2.1.2.4.10.6).

Eq. A.11 is a discrete-time difference equation that corresponds to the solution of an associated underlying continuous-time state variable differential equation (describing the system) of the form:

$$\frac{dx}{dt} = F(t)x + w'(t), \qquad \text{(A.13)}$$

(with details of the above asserted correspondence being provided in [Sec. II]{Kerr93}), where the *transition matrix* for the general time-varying case of *F(t)* is obtained by integration of the homogenous part of Eq.~2.16 over the time interval of interest prior to the next available measurement to be used by the filter. If *F(t)* is constant[54], then the appropriate transition matrix simplifies to just an evaluation of the fairly well-known matrix exponential as

$$\Phi(k+1,k) = e^{F\Delta}, \qquad \text{(A.14)}$$

where Δ is the appropriate time-step between measurements. Similarly, the appropriate discrete-time process noise covariance intensity level, Q_k, to use in the KF mechanization equations corresponding to Eq.~2.16 is obtained by integration of the continuous-time process noise covariance intensity level, $Q_c(t)$, associated with the continuous-time white Gaussian noise *w'(t)* of Eq.~2.16 as [p. 171, Eq. 4-127b]{Maybeck}:

$$Q_k = \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},\tau)G(\tau)Q_c(\tau)G^T(\tau)\Phi^T(t_{k+1},\tau)\,d\tau, \qquad \text{(A.15)}$$

where $\Delta = t_{k+1} - t_k$.

2.3.A.5 The role of fictitious white process noise covariance "Tuning" in improving Kalman Filter tracking performance

Although exoatmospheric target models (used in NMD target tracking) do not usually include any such actual noise sources, *process noise covariance* Q_k still appears in the filter model merely as a useful contrivance for keeping the filter bandwidth open and responsive to later measurements without closing down and ignoring them (after the filter thinks it has discovered the proper track). This trick of "process noise covariance tuning" that most implementers/practitioners use is in the filter's model of the situation merely to elicit good behavior from the filter so that it responds to change by also following departures from what it has grown to expect of the general trend in the target's behavior even when it is only the recent measurements that indicate such changed behavior.

A.3.A.5 The effect of Earth's oblateness on the effective gravity field

Detailed handling of the effect of an oblate ellipsoidal earth is accounted for using the two earth radii being

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[54] Pointers are offered in [Sec. VI]{Kerr93} on how to obtain a correct transition matrix by the appropriate path for general time-varying *F(t)*.

$$R_e = 6378.135 \text{ km} \tag{A.16}$$

$$R_n = 6356.750 \text{ km} \tag{A.17}$$

And with Flattening Factor Being:

$$f = \frac{R_e - R_n}{R_e} = \frac{1}{298.257} \tag{A.18}$$

and with ellipsoid eccentricity being:

$$\epsilon = \sqrt{1 - \left(\frac{R_n}{R_e}\right)^2} = \sqrt{f(2-f)} = 0.08181919 \tag{A.19}$$

and Geocentric latitude, Φ_c, where

$$\tan\phi_c = \frac{z}{\eta} \tag{A.20}$$

where

$$\eta = \sqrt{x^2 + y^2} \tag{A.21}$$

and

$$\eta^2 + \frac{z^2}{(1-f)^2} = R_e^2 \tag{A.22}$$

or in terms of the more familiar local-level geodetic latitude, Φ, as

$$\tan\phi_c = (1-f)^2 \tan\phi \tag{A.23}$$

$$r = \sqrt{\eta^2 + z^2} = \frac{R_n}{\sqrt{1 - \epsilon^2(\cos\phi_c)^2}} \tag{A.24}$$

$$r = R_e\sqrt{1 - \epsilon^2(\sin\phi)^2}. \tag{A.25}$$

The consequential Gravitational potential[55] of the earth, after expanding the familiar (for spherical earth) potential of V_o=G/r into a series of spherical harmonics to account for the earth being an oblate ellipsoid, is

$$V(r) = \frac{G}{r}\left[1 - \sum_{n=2}^{9}\left(\frac{R_e}{r}\right)^n P_n(\sin A)\right] \quad \text{(A.26)}$$

where P_n is the Legendre polynomial of the first kind. The angle A is the angle between the equatorial plane and the vector, measured from the center of the earth to the target. The origin of the ellipsoid has been defined to make the first harmonic zero in the above series and the remaining requisite coefficients are:

$$C_2 = 1.08248 \times 10^{-3}$$

$$C_3 = -2.562 \times 10^{-6}$$

$$C_4 = -1.84 \times 10^{-6}$$

$$C_5 = -6.4 \times 10^{-8}$$

$$C_6 = 0.39 \times 10^{-6}$$

$$C_7 = -0.47 \times 10^{-6}$$

$$C_8 = -0.2 \times 10^{-7}$$

$$C_9 = 0.117 \times 10^{-6}.$$

For the target's position in an ECI frame, the three components of the earth gravity acceleration vector can be shown to be:

$$\ddot{x}_e = -\frac{Gx}{r^3} g_1 \quad \text{(A.27)}$$

$$\ddot{y}_e = -\frac{Gy}{r^3} g_1 \quad \text{(A.28)}$$

$$\ddot{z}_e = -\frac{Gz}{r^3} g_1 - \frac{G}{r^3} g_2 \quad \text{(A.29)}$$

where

[55] Gravitational Constant: G=3.986005 x 10^{14} $m^3/(sec)^2$.

$$g_1 \triangleq 1-\sum_{n=2}^{9}\left(\frac{R_e}{r}\right)^n C_n P'_{n+1}(\sin A) \tag{A.30}$$

$$g_2 \triangleq \sum_{n=2}^{9}\left(\frac{R_e}{r}\right)^n C_n P'_n(\sin A) \tag{A.31}$$

and where $P'_n(\bullet)$ is the derivative of the Legendre polynomial with respect to its argument. Also

$$r = \sqrt{x^2+y^2+z^2} \tag{A.32}$$

$$\sin A = \frac{z}{r}, \tag{A.33}$$

where (x,y,z) are in the ECI frame.

The following recursion relation holding for the derivatives of Legendre polynomials:

$$P'_{n+1}(\alpha) = (2+\frac{1}{n})\left[\alpha P'_n(\alpha) - P'_{n-1}(\alpha)\right] + P'_{n-1}(\alpha) \tag{A.34}$$

is useful for the computation of g_1 and g_2 above. Also note that $P'_0(\alpha)=0$, $P'_1(\alpha)=1$, and $P'_2(\alpha)=3\alpha$. To save in computation time, it is common practice to compute only the most significant first term, corresponding to J_2. According to historical Raytheon rationale circa 1981, the effect of J_2 is to change the impact point by 10 nmi for typical ICBM trajectories. Use of spherical harmonics higher than J_2 contributes less than 0.1 nmi error in impact so they can safely be ignored without significant adverse consequence.

An alternative approach, attributed to Ken Britting within [p. 67]{Regan}, instead uses the ECI frame and only the effect of including the most significant term

$$J_2 = 0.00108263 \tag{A.35}$$

to yield the following values for the three components of gravity acting on the target being:

$$g_x = -\frac{\mu}{r^2}\left\{1+\frac{3}{2}J_2\left(\frac{r_e}{r}\right)^2\left[3-5\left(\frac{z}{r}\right)^2\right]\right\}\frac{x}{r} \tag{A.36}$$

$$g_y = -\frac{\mu}{r^2}\left\{1+\frac{3}{2}J_2\left(\frac{r_e}{r}\right)^2\left[3-5\left(\frac{z}{r}\right)^2\right]\right\}\frac{y}{r} \tag{A.37}$$

$$g_z = -\frac{\mu}{r^2}\left\{1+\frac{3}{2}J_2\left(\frac{r_e}{r}\right)^2\left[3-5\left(\frac{z}{r}\right)^2\right]\right\}\frac{z}{r}, \quad (A.38)$$

where

$$\mu = 3.986005 \times 10^{14}\,\frac{m^3}{s^2}. \quad (A.39)$$

2.3.A.7 {Analytical Bookkeeping Between Necessary Coordinate Transformations}

Additionally, the earth rotates around its polar axis with an angular rate of

$$\omega_e = 7.29211515 \times 10^{-5} radians/sec, \quad (A.40)$$

as used in an Earth Centered Earth Fixed (ECEF) frame. Inclusion of just the effect of J_2 and the omitting of J_3 and J_4 is also stated to be justified for most reentry trajectory modeling applications on [p. 67]{Regan}.

The representation for expressing target motion in going from the Earth Centered Inertial frame (ECI), designated with a subscript I, to the Earth Centered Fixed frame (ECF) that rotates with the earth, denoted with subscript F, is:

$$\begin{bmatrix} x_F \\ y_F \\ z_F \\ \dot{x}_F \\ \dot{y}_F \\ \dot{z}_F \end{bmatrix} = \begin{bmatrix} \cos\omega_e t & \sin\omega_e t & 0 & 0 & 0 & 0 \\ -\sin\omega_e t & \cos\omega_e t & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\omega_e \sin\omega_e t & \omega_e \cos\omega_e t & 0 & \cos\omega_e t & \sin\omega_e t & 0 \\ -\omega_e \cos\omega_e t & -\omega_e \sin\omega_e t & 0 & -\sin\omega_e t & \cos\omega_e t & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_I \\ y_I \\ z_I \\ \dot{x}_I \\ \dot{y}_I \\ \dot{z}_I \end{bmatrix} \quad (A.41)$$

The transformation of target motion in going from the Earth Centered Fixed frame (ECF), designated with a subscript F, to Radar local level frame consists of the following six steps:

1. Rotation by angle λ about the z axis;
2. Rotation by angle Φ_c about the y axis;
3. Translation by the local earth radius along the x-axiz;
4. Rotation by angle (Φ -Φ_c) about the y-axis;
5. Translation by h along the x-axis;
6. Corordinate interchange to obtain the desired sequence of East, North, and Up as, respectively, shown below:

1. $B = \begin{bmatrix} \cos\lambda & \sin\lambda & 0 \\ -\sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix}$;

2. $C = \begin{bmatrix} \cos\phi_c & 0 & \sin\phi_c \\ 0 & 1 & 0 \\ -\sin\phi_c & 0 & \cos\phi_c \end{bmatrix}$, where $\tan\phi_c = (1-\epsilon^2)\tan\phi$;

3. $\underline{r}_e = \begin{bmatrix} -R_{eh} \\ 0 \\ 0 \end{bmatrix}$, where $R_{eh} = \frac{R_n}{\sqrt{1-\epsilon^2\cos^2\phi_c}}$

4. $D = \begin{bmatrix} \cos\phi-\phi_c & 0 & \sin\phi-\phi_c \\ 0 & 1 & 0 \\ -\sin\phi-\phi_c & 0 & \cos\phi-\phi_c \end{bmatrix}$,

5. $\underline{h} = \begin{bmatrix} -h \\ 0 \\ 0 \end{bmatrix}$;

6. $G = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

Using subsrcript R to refer to the radar frame, then using the above intermediate transformations as stepping stones, the relationship can be expressed as

$$\underline{r}_R = (GDCB)\underline{r}_F + G(\underline{h} + D\underline{r}_e) \qquad \text{(A.42)}$$

and after the internal matrix multiplies are performed, the resulting relationships are of the form

$$\underline{r}_R = \mathcal{A}\underline{r}_F + \underline{b}, \qquad \text{(A.43)}$$

where

$$\mathcal{A} = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix} \qquad \text{(A.44)}$$

and

$$\underline{b} = \begin{bmatrix} 0 \\ R_{eh}\sin(\phi-\phi_c) \\ -R_{eh}\cos(\phi-\phi_c) - h \end{bmatrix}; \qquad \text{(A.45)}$$

the last being observed to be a time-independent translation (and hence does not enter into the velocity transformation). Therefore, to go from ECF to radar XYZ, we have

$$\dot{\underline{r}}_R = \mathcal{A}\dot{\underline{r}}_F \tag{A.46}$$

$$\begin{bmatrix} x_R \\ y_R \\ z_R \\ \dot{x}_R \\ \dot{y}_R \\ \dot{z}_R \end{bmatrix} = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 & 0 & 0 & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi & 0 & 0 & 0 \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 0 & -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ 0 & 0 & 0 & \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix} \begin{bmatrix} x_F \\ y_F \\ z_F \\ \dot{x}_F \\ \dot{y}_F \\ \dot{z}_F \end{bmatrix} + \begin{bmatrix} 0 \\ R_{eh}\sin(\phi-\phi_c) \\ -R_{eh}\cos(\phi-\phi_c) \\ 0 \\ 0 \\ 0 \end{bmatrix} . \tag{A.47}$$

New results are offered in {Olson} (c.f., {Nash}).

Now to go between Radar XYZ and RAE (Range, Azimuth, Elevation):

$$R = \sqrt{x^2+y^2+z^2} \tag{A.48}$$

$$A = \arctan\left(\frac{x}{y}\right), \text{ for } 0 \le A \le 2\pi \tag{A.49}$$

$$E = -\arcsin\left(\frac{z}{R}\right), \text{ for } -\frac{\pi}{2} \le E \le \frac{\pi}{2} \tag{A.50}$$

In the above, the proper quadrant must be determined for A so that it only ranges from 0 to 2π.

Seeking the time derivatives of R, A, and E yields

$$\dot{R} = \frac{x\dot{x}+y\dot{y}+z\dot{z}}{R} \tag{A.51}$$

$$\dot{A} = \frac{(y\dot{x}-x\dot{y})}{x^2+y^2} \tag{A.52}$$

$$\dot{E} = \frac{\dot{z}R-z\dot{R}}{R\sqrt{x^2+y^2}} \tag{A.53}$$

In going from RAE to Radar XYZ:

$$x = R\sin A\cos E \tag{A.54}$$

$$y = R\cos A\cos E \tag{A.55}$$

$$z = R \sin E. \qquad (A.56)$$

A2.A.8 Accounting for up or down chirped radar signals typically used in UEWRs (to compensate for Range-Doppler coupling)

When radar measurement of target position is made by means of a linear FM (chirp) waveform, there is a range-Doppler coupling effect. When the target has a non-zero range-rate, the pulse return from the target experiences a Doppler shift that must be compensated in order to use the total round-trip delay to estimate the distance to the target. This Doppler shift induces a range error that is proportional to the range rate, uncompressed pulse duration, and carrier frequency, and inversely proportional to the chirp bandwidth. Since the true range rate is unknown, the effect of this range-Doppler coupling is only approximately canceled through the use of the current best estimate of the range rate as provided by the track estimation algorithm being used. Reference {Fitzgerald}, uses a little correction factor of the constant time step, $\Delta = f_0 T/B$ [56] placed in the observation matrix, H, to pick off or multiply the velocity estimate to also contribute to the position estimate to better reflect the actual physical structure present. Also see {DaumDaum}. In lieu of using separate verify pulses to confirm a target's presence (after having initially detected something), BMEWS now uses a pulse pair, where the first is an up chirp and the second is a corresponding down chirp.

2.3.A.9 ECI referenced target motion model and (R,A,E) radar referenced Measurements}

Reference {Mehra} offers a good discussion of how to formulate the problem of radar tracking of targets in ballistic trajectories and provides a derivation of the particulars of the appropriate mathematical model from first principles, as well as providing an accounting and motivation for use of the various necessary coordinate systems. Other important analytic modeling considerations underlying a rigorous analysis are treated in {Miller3} regarding use of either a ground based or airborne radar for tracking. We used these earlier results as we selected a mathematical model to be used here (as in {Kerr?}).

In our investigation, a Keplerian trajectory is introduced within a detailed simulation of the exoatmospheric target motion to include the effect of an inverse square pull of gravity. We must refrain from just the use of simplified covariance analysis (essentially corresponding to evaluation of a Cramer-Rao lower bound for the estimation objective in the exoatmospheric regime of no process noise being present, as used in earlier investigations {Miller}, {Millerb}) and we instead now incorporate full nonlinear filtering techniques (and the associated standard approximations). Instead of linearizing about the true target, as done in prior simplified covariance analysis, the Extended Kalman Filter linearizes about the filter state estimates at each time-step[57].

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[56] Present day Raytheon implementation now uses a more exacting accounting of the contributing effect in terms of its constituent factors of f_0 being the center frequency, T being the pulse duration, and the swept bandwidth $B=f_2-f_1$ and accounts for both in-plane and out-of-plane components of velocity in a RVCC {Daum3} coordinate system for the target model (G264302-2, Part I, 07 May 1993).

[57] In {Wishner}, the target complex tracking problem is decomposed into primary and secondary contributing effects to be considered in the modeling, and the effect of a rotating earth on the overall problem is of the later category. Our

We work with a fully nonlinear 6-state system model, which in continuous-time is of the form:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ \frac{-\mu\, x_1}{(\sqrt{x_1^2+x_2^2+x_3^2})^3} \\ \frac{-\mu\, x_2}{(\sqrt{x_1^2+x_2^2+x_3^2})^3} \\ \frac{-\mu\, x_3}{(\sqrt{x_1^2+x_2^2+x_3^2})^3} \end{bmatrix} \triangleq f(x)\ , \qquad \text{(A.57)}$$

where μ is the familiar gravitational constant earth mass product GM. This is one of the equations that had to be linearized in implementing an Extended Kalman Filter and for which a Jacobian for the nonlinearity on the right hand side of Eq.~3.15 must be calculated.

Explicit evaluation of the requisite Jacobian, obtained by performing the indicated differentiations on the system nonlinearity *f(x)*, yields:

$$A(x) = \frac{\partial f(x)}{\partial x} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & a_{43} & 0 & 0 & 0 \\ a_{51} & a_{52} & a_{53} & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & 0 & 0 & 0 \end{bmatrix}\ , \qquad \text{(A.58)}$$

goal here of investigating filter tracking performance in an exoatmospheric regime initially dispenses with use of the rotation of the earth (and as a consequence ECI is identical to an Earth Centered Moving (ECM) frame, with the earth rotation rate intentionally taken here to be zero for convenience and as a planned software validation benchmark). In the ~ 30 minutes of an ICBM/SLBM trajectory evolution, the earth rotation doesn't alter the accuracy in EKF tracking performance. Additional realism should be introduced in a controlled quantized manner, where software implementations are demonstrated to work first for a mathematical model devoid of earth rotation; then in a later phase the same software is shown to produce identical performance/output for ECM transformations that are introduced (but with the rotation parameter zeroed) as a logical step in the validation; and, finally, as the last step (not shown here) the rotation parameter of 360°/24 hrs is introduced as part of this standard bootstrapping software validation approach to increased complexity and realism.

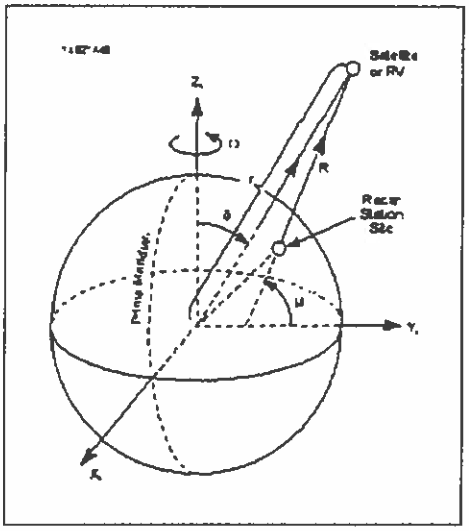

Figure A.3: Earth Centered Inertial (ECI) Coordinates Used in this Formulation (figure is from [10], [40]).

where in the above

$$a_{41} = \frac{\mu[2x_1^2 - x_2^2 - x_3^2]}{(x_1^2 + x_2^2 + x_3^2)^{5/2}}; \quad a_{42} = \frac{+3\mu x_1 x_2}{(x_1^2 + x_2^2 + x_3^2)^{5/2}}; \quad a_{43} = \frac{+3\mu x_1 x_3}{(x_1^2 + x_2^2 + x_3^2)^{5/2}}, \tag{A.59}$$

$$a_{51} = \frac{+3\mu x_2 x_1}{(x_1^2 + x_2^2 + x_3^2)^{5/2}}; \quad a_{52} = \frac{\mu[2x_2^2 - x_1^2 - x_3^2]}{(x_1^2 + x_2^2 + x_3^2)^{5/2}}; \quad a_{53} = \frac{+3\mu x_2 x_3}{(x_1^2 + x_2^2 + x_3^2)^{5/2}}, \tag{A.60}$$

$$a_{61} = \frac{+3\mu x_3 x_1}{(x_1^2 + x_2^2 + x_3^2)^{5/2}}; \quad a_{62} = \frac{+3\mu x_3 x_2}{(x_1^2 + x_2^2 + x_3^2)^{5/2}}; \quad a_{63} = \frac{\mu[2x_3^2 - x_1^2 - x_2^2]}{(x_1^2 + x_2^2 + x_3^2)^{5/2}}. \tag{A.61}$$

In order to use the above ECI coordinate frame and system equations, the measurement equations are of a form addressed below. The measurement equations used for the present sensor model are as obtained from Figs. 27 and 28.

The resulting sensor measurements in terms of range, R, and the direction cosines, u and v, to the target PVB are:

$$R = \sqrt{x'^2 + y'^2 + z'^2}, \tag{A.62}$$

$$u = \frac{x'}{R}, \tag{A.63}$$

$$v = \frac{y'}{R}, \tag{A.64}$$

where x', y', and z' are as in Fig. 28 (and are to be defined next).

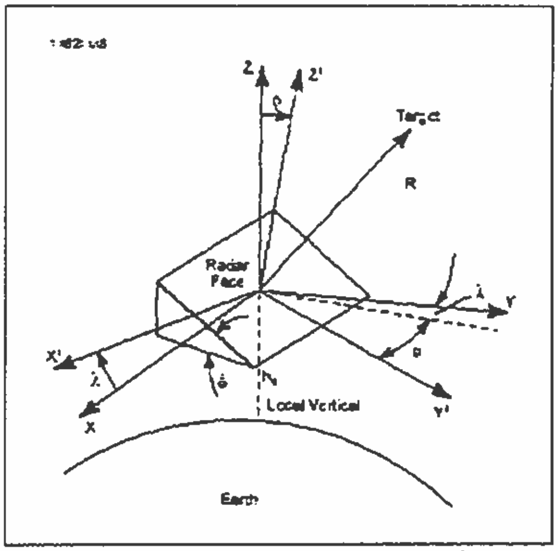

Figure A.4 Sensor Measurements Must now be Referenced to Earth Centered Inertial (ECI) Coordinates Used in thei Formulation (figure is from [10], [40]).

In Fig. A.4, the local coordinates *x, y, z* are located at the center of the sensor face in the plane of the array. In this coordinate system, *z* is directed along the local vertical and *x* and *y* lie in the horizontal plane, with *x* pointing East and *y* pointing North. From [Section II]{Mehra}, these local level coordinates *x, y, z* can be re-expressed in terms of *x', y', z'* coordinates, via the following transformation

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = T \begin{bmatrix} x \\ y \\ z \end{bmatrix},$$

where

$$T = \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 \\ \cos\phi\sin\lambda & \cos\phi\cos\lambda & -\sin\phi \\ \sin\phi\sin\lambda & \sin\phi\cos\lambda & \cos\phi \end{bmatrix},$$

as the appropriate change of coordinates corresponding to the rotation depicted in Fig. 28, where the above parameters of λ and Φ, are also defined in Fig. 28. The coordinates *x', y', z'* are oriented so that *z'* is normal to the face of the sensor array, and *y'* lies on the face of the array, and *x'* lies along the intersection of the sensor face and the horizontal plane[58].

[58] The mathematics of this transformation is consistent with Fig. 28. R. M. Miller's software implementation code for getting between sensor Face Centered Coordinates to Earth Centered Inertial coordinates {Miller3} avoids sinusoids

The above received sensor signal-processed measurement can be reexpressed in terms of the measurement of target range (as appropriate for a radar or other active sensor if not range-denied due to jamming), elevation, and azimuth as, respectively:

$$r = \sqrt{x^2 + y^2 + z^2}, \quad (A.65)$$

$$E = arctan\left[\frac{z}{\sqrt{x^2 + y^2}}\right], \quad (A.66)$$

$$A = arctan\left[\frac{x}{y}\right], \quad (A.67)$$

where the length in Eq.~31 is identical to the length in Eq.~28 since the transformation T is a rotation (and as such is an orthogonal transformation which preserves lengths). The expressions of Eqs.~31 to 33 correspond to the following measurement equation:

$$\begin{aligned} z(t) &= \begin{bmatrix} r \\ E \\ A \end{bmatrix} + v(t) \\ &= \begin{bmatrix} \sqrt{x^2 + y^2 + z^2} \\ arctan\left[\frac{z}{\sqrt{x^2+y^2}}\right] \\ arctan\left[\frac{x}{y}\right] \end{bmatrix} + v(t), \end{aligned} \quad (A.68)$$

where the Gaussian white measurement noise, $v(t)$, has a covariance that is of the form[59]

$$R = \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_E^2 & \\ 0 & 0 & \frac{\sigma_E^2}{cos^2(E)} \end{bmatrix}, \quad (A.69)$$

and the proper values to use for these variances are provided with our numerical results in Secs. 2.2.3.7 and B.7.

An additional aspect not to overlook is that target location is referred back to ECI coordinates within the software by subtracting out the known location of the stationary radar array as an offset or translation. Notice that for the above described target complex motion model of Eqs.~23 and 34,

within the transformation by resorting instead to the underlying right triangles corresponding to each angle measurement. This alternative implementation appears to offer some nice efficiencies so we also employ it here in our investigation.

[59] Within the software, we intentionally avoid the presence of the cosine in the denominator depicted in Eq.~35 by instead employing the following identity: $\cos^2 E = \frac{x^2 + y^2}{x^2 + y^2 + z^2}$.

respectively, both the system model *and* the measurement model are nonlinear. The linearization of the above nonlinear measurement of Eq.~34 is as provided below (from [pp. 22, 23]{Miller3}):

$$\frac{\partial r}{\partial x_o} = \frac{x}{r}; \quad \frac{\partial r}{\partial y_o} = \frac{y}{r}; \quad \frac{\partial r}{\partial z_o} = \frac{z}{r} \tag{A.70}$$

$$\frac{\partial E}{\partial x_o} = \frac{-xz}{\rho r^2}; \quad \frac{\partial E}{\partial y_o} = \frac{-yz}{\rho r^2}; \quad \frac{\partial E}{\partial z_o} = \frac{\rho}{r^2} \tag{A.71}$$

$$\frac{\partial A}{\partial x_o} = \frac{y}{\rho^2}; \quad \frac{\partial A}{\partial y_o} = \frac{-x}{\rho^2}; \quad \frac{\partial A}{\partial z_o} = 0 \tag{A.72}$$

The linearization of the current EKF is about the most recent state estimate $\hat{x}_{k|k}$ instead of the actual state (which is realistically presumed to be unknown to the observing sensor but is treated as known for CR lower bound evaluation).

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