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New version read in by optical scan and converted to Latex format. Used WP spelling checker to correct Sp's. Current version old here still has misspellings, typos: please excuse.

KALMAN FILTER TRACKING IN THE MULTITARGET CASE AND AN ASSOCIATED PRACTICAL SONOBUOY SELECTION PROCEDURE

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ABSTRACT

An overview perspective is provided on the various alternative approaches to multitarget tracking that have emerged in the last 20 years. The salient characteristics of each is summarized as benefits and drawbacks are enumerated and cross-comparisons are made. Open questions are indicated that remain unanswered. While the original multitarget investigations were mainly developed for radar applications in Ballistic Missile Defense (BMD), this investigation considers the matter from the viewpoint of a passive sonobuoy application. Along these lines, an associated simple algebraic sonobuoy selection algorithm is described that enhances the accuracy of target location results while reducing the total signal processing burden by limiting the number of sonobuoys to a select few that need be fully processed.

1. INTRODUCTION/OVERVIEW

A brief summary perspective is offered in Section 2 of several existing theoretical approaches that have been developed for handling the more operationally realistic multitarget case (as contrasted with designs that can only handle the case of tracking a single target), and, in particular, the candidates are narrowed down to the approach that appears to be best suited for sonobuoy tracking applications. The present task is made somewhat easier by the availability of the open literature IEEE surveys of Bar-Shalom [1] in 1976, Reid [2] in 1979, and Chang and Tabazynski [3] in 1984, however, there is still this need to follow-up and cross-check in providing the results since each previous survey is now either somewhat outdated, or somewhat self-serving, with emphasis provided mainly in its own particular application area of primary concern being principally radar oriented for Ballistic Missile Defense (BMD). In contrast, submarine and sonobuoy tracking [45] are of primary interest here. To this end, a high level technical summary of the salient characteristics of each approach is provided in Section 2 along with an indication of its advantages for the sonobuoy tracking application as well as its disadvantages so that inherent limitations are not overlooked.

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Based on perceived similarities between post-coherence function target localization and principles of operation of hyperbolic LORAN radio navigation systems, a novel approach is offered in Section 3 for selecting sonobuoys for participating in target localization computations. This sonobuoy selection approach simultaneously offers not only improvement in the quality of the target localization solution by enforcing good physical geometry but also offers a reduction in overall computational burden over what would otherwise be present without it. The rationale motivating the technique is also provided. A summary is provided in Section 4.

2. SUMMARIZING AND WEEDING OUT ALTERNATIVE APPROACHES FOR HANDLING MULTITARGET TRACKING

Morefield [4] rigorously poses the multitarget tracking problem as a structured multihypothesis "Bayesian" test, which is performed to determine which particular combination of the many feasible tracks is most likely to represent actual targets. The approach of [4], which may be classified under the category of supervised pattern recognition, embodies:

1. Estimating the number of actual tracks (data clusters) present;

2. Estimating the parameters of individual trajectories;

The structural form of the underlying state space representation of targets (described merely by Newton's Second Law with drag or viscous effects included when warranted in the particular application) and the associated quantification of process and measurement noise statistics are taken to be known as a physically realistic assumption made within the methodology prescribed in [4]. The problem is to determine which of the targets is actually being observed by the sensor (or sensors) as each measurement is obtained. This problem is aggrevated by the confusion caused by:

1. Possible presence of closely-space targets (as in convoys or in close trailing);

2. Possible periods of poor viewing geometry;

3. Possible periods of high intensity sensor noise or background noise (as with thunder storms, schools of whales, heavy commercial shipping lanes nearby);

4. Possible signal fades due to uncertainties in the acoustic transport medium;

5. Possibly inadequate sample rate at times due to gaps (as from missing sensor data reports for whatever reason);

6. Possible presence of false alarms or "phantom targets" as likely to occur in practical detect-first-then-track situations.

Underlined items in the above list are from [4]. The original list is further augmented here for completness and clarity, and to emphasize additional aspects relevant to sonobuoy applications.

2.1 A NOVEL ZERO-ONE INTEGER PROGRAMMING FORMULATION

The discrete-time version of the decision-directed multiple hypothesis testing situation, as depicted in [4], was further demonstrated there to exactly correspond to the mathematical problem of solving a particular zero-one (0-1) integer programming problem, which involves minimizing a linear cost function of the form:

$$J[\underline{\theta}] = \underline{c}^T\underline{\theta} \tag{2-1}$$

subject to linear equality constraints of the form

$$A(\underline{\theta}) \leq [1, 1, 1, \ldots, 1]^T \tag{2-2}$$

but where only vectors $\underline{\theta}$ with binary component entries being either zeroes or ones are admissible as solutions.

The cornerstone of the match-up in [4] between the underlying multihypothesis testing for the multitarget application with the solution procedure of zero-one integer programming involves operating on the following probability density function describing the entire set of sensed measurements:

$$p(\underline{Z}|\tau) = \prod_{\lambda^j \epsilon \tau} p(\lambda^j|\tau) \tag{2-3}$$

where

$\underline{Z} \overset{\Delta}{=}$ set of all measurements collected and is partitioned to correspond to m tracks $\lambda^i (i=1, \ldots, m)$, where m may vary from one hypothesis τ to another,

(and π is the symbol for products of these probability density functions as indexed over each λ^j within τ). Thus selecting (i.e., choosing) τ to maximize the following density function:

$$\max_{\tau \epsilon F} p(Z|\tau) = \max_{\tau \epsilon F} \prod_{\lambda^j \epsilon \tau} p(\lambda^j|\tau)P(\tau) \tag{2-4}$$

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where

$F \stackrel{\Delta}{=}$ feasible set (of cardinality r),

$P(\tau) \stackrel{\Delta}{=}$ constant* (i.e., uniformly distributed in lieu of no other *a priori* information being provided on which hypothesis is more likely to be correct),

can be demonstrated to be equivalent (in the more realistic case acknowledging the possible occurance of <u>some</u> false alarms) to:

$$\max_{\tau \epsilon F} [\sum_{\lambda^j \epsilon \tau} \{\ln p(\lambda^j|\tau) + n_j \ln Y\}] \tag{2-5}$$

where

$n_j \stackrel{\Delta}{=}$ number of measurements in track λ^j,

$Y \stackrel{\Delta}{=}$ size of the surveillance area.

Finally, via the observations of [4], [5], [6], the optimization problem of Eq. 2-1 is recognized to be equivalent to:

$$\max_{\theta_\tau \epsilon T} [\sum_{j=1}^{r} \{\ln p(\lambda^j|T) + n_j \ln Y\} \theta^j{}_\tau] \tag{2-6}$$

by defining the scalar c_j weightings occuring in the cost function of Eq. 2-1 to be:

$$c_j \stackrel{\Delta}{=} -\ln p(\lambda^j|\tau) - n_j \ln Y \tag{2-7}$$

with

$T \stackrel{\Delta}{=}$ set of all binary vectors of $\theta_\tau = [\theta^1{}_\tau, \theta^2{}_\tau, \ldots, \theta^r{}_\tau]^T$ corresponding to the set of all subsets of F,

and

$$\theta^j{}_\tau \stackrel{\Delta}{=} \begin{cases} 1 & \text{if } \lambda^j \epsilon \tau \\ 0 & \text{otherwise} \end{cases} \tag{2-8}$$

Therefore, in the above, the multihypothesis measurement classification problem is revealed in Eq. 2-6 to be exactly the form of a zero-one integer programming problem as depicted in Eq. 2-1 and

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* It is stated on p. 618 of [1] that, strictly speaking, the approach of Morefield [4] is actually <u>not</u> totally Bayesian because [4] only utilizes a priori values of $P(\tau)$=constant.

2-2. Thus Morefield [4] rigorously demonstrates that the multitarget tracking problem may be reduced to a relatively well-studied mathematical problem in discrete optimization theory classified as the so-called "set partitioning and set packing problems of 0-1 integer programming"*. From further structural considerations of what operations of the solution algorithms are more efficiently mechanized as parallel computations, certain practical aspects of solution algorithm implementation have already been worked out by Morefield [7].

The above described multitarget tracking problem, after having now been reduced to a zero-one integer programming problem, can be solved by any one of several alternative solution algorithms that are further generalizations of the Munkre's "assignment" algorithm [8] (requiring at most $(11N^3+12N^2+31N)/6$ operations for problems of dimension N) rather than an exhaustive search of N! possibilities, or as the generalized Hungarian algorithm (as put in perspective in [9] for a proper appreciation, as endorsed by MIT's Prof. Munkre in private communication) with specific integer programming specialization having been provided in [10]. Comparisons were made in 1979 between alternative solution procedures when evidence was amassed from computational runoffs by Narula and Kindorf in [11] to enable recommending either:

1. Zoints' generalized additive algorithm (as the primary candidate);

2. Balas' additive algorithm (as a close runner-up);

as the preferred mechanisms for implementation over four other well-known algorithms also used to solve 0-1 integer programming problems. These two algorithms were recommended over the other four following relative considerations in [11] of:

1. Accuracy in homing in on the correct answer,

2. Speed of convergence,

3. Efficiency in mechanization as entailing the least computational burden, thus requiring the least CPU time.

Seeking better, more efficient algorithms for solving 0-1 integer programs continues to be an active topic of research and has been discussed at several recent meetings of the Operations Research Society of America/The Institute of Management Science (ORSA/TIMS).

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* This is the name used for this type of problem by mathematicians, operations research practioners, management science/economists, and by many cognizant engineers.

2.2 A CONTROVERSY AND ITS RESOLUTION

To demonstrate that a brooding controvery is not being overlooked here, an open literature objection to [4] and its successful resolution are briefly summarized. Unfortunately, as with many other page-constrained open literature discussions, some slight ambiguity occurred in the original description provided by [4] in sketching the major technological steps of the approach. In scrutinizing and reconstructing the bridging material betwen the steps offered in [4], an indictment was subsequently made by Bailey in [5] that the integer programming formulation of Morefield [4] had overlooked the fact that the vector $\underline{c}$ in the cost function of Eq. 2-1 is actually (as claimed in [5]) a function of the variable θ to be minimized with respect to and that the resulting cost function, instead of being like Eq. 2-1 is actually of the form:

$$J[\theta] = \underline{c}^T(\underline{\theta})\underline{\theta} \tag{2-9}$$

which (by not properly conforming to the specific structure of being a constant vector $\underline{c}$) is no longer a provably tractable 0-1 integer programming problem. In time, this charge from Bailey [5] was dismissed when the original problem formulation as offered in [4], was eventually vindicated by Lehtomaki in [6] through an independent investigation (funded by the Office of Naval Research) which adequately resolved those points of discrepancy claimed in [4].

However, based on prior experience [47], [48], a less serious weak spot is perceived here to persist in the methodology described in Morefield regarding the statistical computations offered in evaluating false alarm probabilites in [4] based on the whiteness of the associated Kalman filter residuals. Such are anticipated here to not really be totally white because:

1. The state-space equations underlying the application are not strictly linear, especially for the anticipated sonobuoy application;

2. Even if they were close to linear or had been successfully linearized, the reduced-order dimension of the practical filter implementation is usually much less than what actually occurs as the dimension of the theoretical model so the resulting residuals are not totally white and unbiased.*

However, it should be possible to derive and computationally evaluate bounds (similar to [12], [13] [46], [49]) for these requisite probabilities now that these departures from the ideal situation are honestly acknowledged, as clarified herein.

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* A criticism has recently been levied in the first paragraphs of Section 4 in [14] and in Section 2 of [15] relating to the deleterious impact of an assumption of white noise residuals for practical reduced-order Kalman filter implementations on the ultimate performance of several recent approaches to failure detection in dynamic systems.

2.3 FURTHER CROSS-COMPARISONS OF ALTERNATIVE APPROACHES

We could stop here on the subject of multitarget tracking since we have presented the essence of the approach of Morefield [4] that we feel consistutes the best algorithm for handling the sonobuoy applications. However, lest we be faulted for not selecting subsequent so-called improved algorithms with more "bells and whistles" (and greater associated computer burden) than [4], we proceed to show how this algorithm stacks up against both prior and later approaches to multitarget tracking.

As identified by Bar-Shalom on p. 621 of [1], once the most likely set of tracks has been selected by solving the integer programming problem formulation of [4] described above, the state estimates and covariances are computed from a corresponding set of standard Kalman-like filters, and it is observed here that either repeated instantiations of the same Kalman filter routine, or even better, only repeated calls to the Kalman filter subroutine might suffice for each designated target being tracked. However, being limited to only repeated calls to a Kalman filter subroutine without some type of multitarget framework as a super structure (such as that provided in [4]) probably is not completely satisfactory for sonobuoy processing where multiple targets usually occur in practical situations. To prevent a growing computational burden such as that warned of at the end of the second paragraph on p. 620 of [1], it is suggested here that a fixed limit can be enforced on the number of Kalman filters enabled or, equivalently, limits may be imposed on the allowable number of subroutine calls within a designated time epoch (corresponding to the total number of tracks to be followed-up on as being feasible to handle within the hardware/software resources available). Another perceived drawback in the method of [4], as identified by Bar-Shalom in [1], is asserted to be that the probability of detection is routinely assumed to be unity. However, in his first footnote on p. 620 of [1] it is further elaborated that a non-unity detection probability may be easily accomodated by merely allowing one additional split track option at each measurement sampling or measurement sensor reporting time. Thus Bar-Shalom offers a nice resolution to his own objections to the approach of Morefield.

For additional perspective on the ultimate utility of the insights offered by Bar-Shalom in [1], it is mentioned here that the simple approach for handling the occurrance of missing data or gaps in the measurements as offered by Jaffer and Gupta in [16] is discussed and endorsed in [1]; however, the fundamental weaknesses of [16] are apparently not recognized by Bar-Shalom in [1] despite the severity of impact. The issues are clarified by Tugnait and Haddad in [36] as a fundamental survey and update on how to rigorously handle the occurance of gaps in measured data within the context of Kalman filtering. Such issues are of utmost importance for almost autonomous real-time processing without the benefit of human intervention and deciphering at intermediate stages as is anticipated to occur in handling the measurement returns for multiple sonobuoys otherwise too much delay associated with human response time would be incurred. Also, human operator participation should be for exception handling so that they are not swamped with almost meaningless routine reviews of massive amounts of processed data.

At the other extreme is how to handle the case of obtaining several possibly anomolous returns in the vicinity of a single designated target. The approach of Sittler [17] was developed so that measurements of uncertain origin could be reasonably incorporated into an existing tracker (where [17] utilized the imposed tracking methodology in vogue prior to widespread adoption of Kalman filter trackers). Sittler [17] proceeded to split the track (by acknowledging the possibility of several distinct closely spaced targets) whenever more than one return (detection) was received from within a prescribed neighborhood of the "predicted" measurement occurence. The approach of Fraser and Meier [18] was similar to that of Sittler [17], but utilized the now standard Kalman filtering context for tracking in its active sonar application. The fully optimal approach of Singer, Sea, and Housewright [19] involved excessive splitting of tracks which were extended all the way back in time to the initial measurement returns received at algorithm start-up and subsequent reassignment and recombining of the totality of measurement results received since initialization. This resolve of the algorithm to start from scratch could occur at any point of uncertainty in measurements received. To avoid the otherwise horrendous exponentially unbounded increase in computer burden required in an implementation of this optimal algorithm, optimality in performance is traded-off for a fixed computer burden by only allowing the algorithm to make a reassignment of measurement associations over the last few measurements received within a sliding time window (of fixed, reasonable duration). Thus reasonable performance can be obtained in a practical mechanization of this type of variant of [19].

According to Bar-Shalom [1], variations on the approach subsequently pursued in Morefield [4] exist (e.g., [20]), as do variations of the approach of Singer, Sea and Housewright [19] (which assumed just one target and the presence of several inconsistent measurement returns) as in Bar-Shalom and Tse [21] (with one target, multiple returns observed) further extensions in Bar-Shalom [22] (with several targets and an arbitrary number of returns), and Alspach [23] (with fixed m-targets and exactly m returns).

According to Morefield [4] (p. 303), the approaches of [22] and [23] avoid the combinatorial problem and merely form locally optimal trajectory estimates using data from several benignly contiguous tracks rather than tackling the more challenging problem, where the measurement data must be partitioned into individual track assignments when physical conditions (frequently) are severe enough to warrant this more exhaustive approach (as utilized in [4] and [2]). The approach of [4] does offer a reasonable compromise between one extreme of using an excessively simplistic approach (which avoids or ignores the naturally occurring combinatorics entirely) and the other extreme of computationally accounting for <u>all</u> of the different ways n measurements can be partitioned into m tracks, which is (p. 303 of [4], [24]):

$$\frac{1}{m!} \sum_{i=1}^{m} \binom{m}{i} (-1)^{m-i} i^{n} \qquad (2\text{-}10)$$

different ways!

Reid [2] handles the problem of tracking several targets of interest and <u>simultaneously</u> handles the problem of appropriate track initiation. Reid offers an overview of his algorithm depicted here as Figure 1, where most of the processing is to occur in the four subroutines depicted. The CLUSTER subroutine associates measurements with previous clusters. If two or more previously independent clusters are associated together because of a measurement, then the two clusters are combined into a "super cluster". A new cluster is formed for any measurement not associated with a prior cluster as is standard practice in radar and ultrasonics processing. As an integral part of the initialization program, previously known (perhaps friendly) targets form their own individual clusters. The HYPERGEN subroutine forms new data-association hypotheses for the set of measurements associated with each cluster. Within the approach of Reid [2], the probability of each such running hypothesis being correct is calculated on-line and target estimates are updated for each hypothesis of each cluster. Based on encounters of low computed probability, the subroutine REDUCE is invoked to either eliminate an unlikely hypothesis or to merge hypotheses with similar target estimates. Once hypotheses have been simplified in this manner, the MASH routine is used to purge from further statistical consideration those measurements that are no longer considered to be ambiguous. Once a measurement achieves this status, a previously tentative target is elevated to the status of a confirmed target (similar to the concept utilized within Naval Tactical Data Systems (NTDS)).

As the basis for an at-a-glance overview comparison, Reid [2] provides a table similar to what is depicted here as Table 1; however, the table here is augmented to include explicit consideration of the approach of Reid [2] itself as an additional column on the far right. Table 1 is also slightly altered to rectify the incorrect situation depicted for Morefield's algorithm [4] in [2] as a perhaps inadvertent set-up since Morefield's approach [4] <u>does</u> offer a recursive filter implementation which handles information on several targets despite what Reid had indicated in his original table in [2] regarding these two issues. Using clusters is one way of handling the problem but it is not the only way to handle it as [4] demonstrates.

In Reid [2], it is remarked that the approach of Sittler [17] was more than a decade ahead of its time in how it identifies and handles all the fundamental items of concern including a reasonble handling of targets that cease to exist (i.e., track termination or track extinction in warfare applications). This last feature is identified as being equivalent to including the concept of a "track's status being good" if data is still being received that served to prevent the track from being dropped as being no longer active. Unfortunately, the approach of [2] is acknowledged to not include this capability. This approach of [4] also involves less of a computational burden than [2], having fewer probability calculations to update since it does not utilize the additional "a posteriori" probability calculations of [19], [21], [2], [3]. The additional computational burden of "a posteriori" probability calculations beyond the relatively simple expressions for a priori calculations provided in the approach of [4] (as already discussed) may not be absolutely necessary for the sonobuoy application and should perhaps be dispensed with. Therefore, the approach of Morefield [4] appears to suffice as a reasonable technique for handling the multitarget case for target tracking applications using sonobuoys. Minor interface engineering may be required for ultimate implementation and tailoring to the sonobuoy application.

It is recognized that at least one proposed approach (e.g., [25], as discussed in Section I.1 of Appendix I in [26]) advocates sonobuoy utilization involving manipulation of the computations corresponding to the processing of data received simultaneously from an aggregate of sensors. Along this line, it is stated in Chang and Youens [27] that [28] offers considerations of efficient algorithms for processing multiple sensor data (only after) completion of:

1. The sorting out and identification of a time sequence of measurements associated with the same target;

2. The sorting out of measurements from several different sensors and linking (or cross-correlating) those that should be associated with the same target.

According to Chang and Youens [27], the objective of the above item 2 as the association of measurements from several different sensors with the current target (denoted in [27] as the "sensor to sensor measurement correlation problem" (of [30])) is equivalent to the classical "assignment problem" solved by the Munkre's/Hunarian algorithm* (as already discussed above). However, [27] asserts on p. 10 and p. 15 that for handling more than two sensors simultaneously, the problem is an obvious reasonable generalization of the "assignment problem" but with no solution algorithm yet available ([3]), while an exhaustive search would be impractical by requiring the enumeration of $(n!)^{M-1}$ possibilities. This, therefore, is an open research problem whose solution should have

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* A suboptimal algorithm for accomplishing an "assignment" type solution called the "row and column elimination" method is also sketched out on pp. 9-1 of [27] and is stated to suffice when the target density is "sufficiently low", but explicit quantification is missing on what target densities are classified as being "sufficiently low".

great utility in allowing sonobuoy returns for more than just pairwise processing in post-coherency function to target localization [37]. [The solution to this problem should also be of interest in BMD and SDI.]

Other issues associated with acoustic tracking filters such as coordinate system selection and how to enhance system observability enough for successful bearings-only filter tracking are treated in [31], as endorsed by [3] with only their radar experience even though the measurement noise covariance matrix is inherently singular in the model defined in Eq. 13 of [31] and is therefore less acceptable* for Kalman filtering. A consideration of the recommended coordinate system and observability needed for sonar tracking is provided in [34].

In contradistinction to the previously discussed algorithms that use a Kalman filter, it is informative to see how pure statistical tests without modeled dynamics perform in tracking a single moving target. Monte-Carlo simulations are utilized in [35] to compare performance results in tracking a <u>single</u> moving target using the following three alternative statistical algorithms:

1. Nonparametric test based on the rank statistic;

2. Parametric test based on the likelihood function;

3. Parametric test based on the sample average.

The last test was recommended in [35] as having the best performance overall considering its modest computational burden for implementation. However, the single target and benign test environment of [35] make it less significant for sonobuoy processing target tracking than the other approaches already discussed above.

A lucrative approach, not yet addressed prior to [38] for the sonobuoy application to our knowledge, is the benefit of coordinating target tracking filters on separate cooperating surveillance platforms using different sonobuoy mixes for additional perspective as a decentralized approach to data fusion for enhanced detection and estimation/tracking of enemy targets. Sonobuoy processing could parallel what is done for radar targets in NTDS [39], where separate platforms can utilize the Grid Lock mode to cross-corroborate radar images from the differing perspectives of separate platforms and to use the friendlies appearing on the radar screen as an anchor to sift out false alarms for a better assessment of the correct number of unfriendlies present by appropriately combining some of the separate blips into one when there is only one hostile target present in that vicinity. The encouraging potential of such an approach is indicated in [40], [41], [58]. Especially encouraging is the fact that decentralized implementations of Kalman filters are already emerging for navigation applications (see Section 4.2 of [15], [44].

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* Relatively unknown techniques for getting around this impediment are available [32], [33] although undesirable to have to use because of a somewhat increased computational burden.

Decentralized filter implementation should perhaps be considered in deciding how to best integrate sonobuoy-based target tracking systems into overall existing or emerging Navy command, control, and communication structures to achieve improved performance while minimizing data processing requirements. It is this final problem that is emphasized on p. 108 of [3] in 1984 as requiring current attention.

3. SONOBUOY SELECTION TO ENHANCE POST-COHERENCE TARGET LOCALIZATION

Conceptually, the process of post-coherence function target localization is as depicted in Figure 2 with the intersection of two (or more) hyperbolas (corresponding to two distinct sensor pair correlations providing constant difference in delay time-of-arrival (see p. 1500 of [37]). This process is analogous to the operation of a hyperbolic LORAN-C radio navigation system (in the standard hyperbolic mode rather than in the rho-rho mode where there would be intersecting circles).

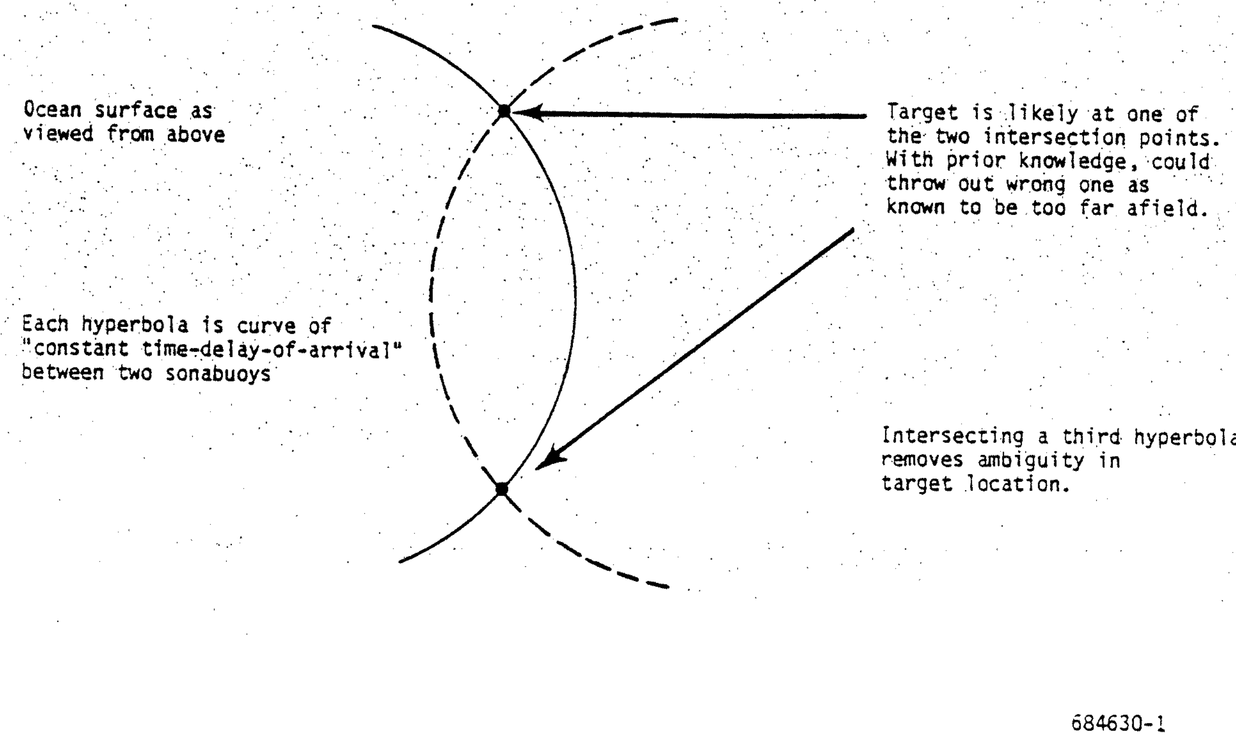

FIGURE 2: Simplified Principles of Sonobuoy Target Localization from Intersection of Hyperbolas of Constant Delay-Time-of-Arrival

However, like LORAN ([42], [43]), post-coherence target localization can also suffer from the effects of bad geometry corresponding to what may be incurred by a LORAN user in encountering bad Geometric Dilution of Precision (GDOP) if he is in an unfavorable location with respect to the fixed LORAN transmitter sites within radio reception range. Moreover, unlike what is simplistically depicted in Figure 2, the problem is not strictly planar for sonobuoy applications* because of: (1) possible ray bending in the rather nonlinear acoustic medium due to the thermal gradient and (2) the fact that when depth is properly considered, the curve of constant delay-time-of-arrival is actually a slightly contorted/distorted hyperbolic surface. Even with these minor complications, several hyperbolic surfaces may be intersected (by intelligent selection of sonobuoy pairs to participate in the processing) to yield a unique solution.

The least sensitivity to error in final target localization solution is obtained when the hyperbolic lines or surfaces intersect at almost 90° angles. This favorable situation occurs when the baselines drawn between the locations of the sonobuoys utilized are approximately orthogonal. Such considerations can be utilized as the theoretical basis of a proposed "Executive Sonobuoy Selection Algorithm" to automatically select (as an operator aid) the subset of available sonobuoys to be used in the target localization process. Such a subset selection would enhance the GDOP associated with calculating the solution as the target location yet avoid any additional less productive sonobuoy processing (as a reasonable way to conserve scarce computational resources for this application). Please notice that the computations to be further described below that constitute the proposed Executive Sonobuoy Selection Algorithm are all simple and may be accomplished from a simplified planar viewpoint in selecting (i.e., recommending to the operator) sonobuoy participation based on most nearly orthogonal baselines (or those having the largest angles but less than 90° between baselines) where baselines are errected between available locations of candidate sonobuoys.

The mechanics of checking to enforce large angles of intersections of the Lines-of-Position (LOP) are relatively straightforward. The location of transmitting sonobuoys can be established even in the presence of significant sea current drift effects by periodic overflights such as is done with the conventional Sonobuoy Reference System (SRS) or through the proposed use of disposable lightweight Global Positioning System (GPS) receivers with radio locators for each buoy.

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* The transmitting medium for LORAN is somewhat more well-behaved, but still suffers from a somewhat heuristic treatment of over-land, over-water, over-ice corrections and atmospheric interference.

Candidate sonobuoys should then be used to form candidate baselines by erecting straight lines between each two transmitting sonobuoys. All candidate sonobuoys should be taken pairwise "two-at-a-time" as selected from all possible candidate sonobuoys so that all candidate baselines are formed. After all candidate baselines are formed, the inner products of each pair of baseline directions divided by the magnitude of the two baselines is the cosine of the angle between them. These angles of intersection should be ranked in magnitude for the selection that takes place next. The larger or closer this intersection is to 90°, the closer the intersection of the corresponding LOPs are to 90° and consequently the better the geometry of position location will be. Narrowing the field of candidates to the particular LOPs to be ultimately utilized should also be associated with proximity to any available indications of observed target locations in order to enhance the intensity of target returns. Thus the sonobuoy baseline quadruples (i.e., two intersecting baselines or equivantly four (or perhaps three where one sonobuoy could be shared) sonobuoys per two LOP intersections) can be ranked on the magnitude of the angle of intersection as it declines from an ideal of 90° as the basis for selecting only the best sonobuoys for further participation.

Employing a procedure such as described above thus specifies those sonobuoys that will offer more utility in localizing the target and serves to identify higher priority sonobuoys to be included for subsequent LOFAR, DIFAR, etc. processing. This is especially important in any situation of scarce computational resources (i.e., saturation in the number of available sonobuoys versus the processing power available to sort things out) so that less fruitful sonobuoys are weeded out from further participation based merely upon their location prior to any signal processing of their outputs.

4. SUMMARY

Several different approaches to multitarget tracking were reviewed in Section 2. Salient characteristics of each approach were presented as cross-comparisons were made between benefits that each had to offer. The review presented here was from the, perhaps, more myopic perspective of relevance to just the particular scenario of sonobuoy processing. Other approaches to the problem that are not discussed here (such as [50] - [57], [59] - [61]) are currently still evolving. This slightly critical review is to serve as a convenient roadmap and/or score card for others concerned with sonar/sonobuoy target tracking in particular or multitarget tracking in general. Surveys of this type are possibly somewhat tainted or loaded when they also seek to have a particular approach which the author pioneered be accepted or adopted as the ultimate refinement in this area to date. The author of this survey has no personally developed approach to multitarget tracking and therefore has no need to "grind his own axe", but merely has experience in Kalman filter applications [14], [44], [45], [47] - [49], so this survey should be devoid of any ego related subjective bias sometimes encountered when an author is trying to push his own approach in making comparisons to other approaches.

In Section 3, a simple technique was described for enhancing the geometry of sonobuoy target localization while also reducing the associated computational burden of sonobuoy processing. This novel idea arose due to the strong similarity between the geometry of LORAN radionavigation positioning and sonobuoy post-coherence function localization. The lure and potential benefits of going to decentralized processing and decision making and exploiting techniques similar to Grid Lock moding in NTDS were also described here for this sonobuoy application.

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