Drawbacks of Residual-Based Event Detectors like GLR or IMM Filters in Practical Situations

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***Abstract*— Use of the Generalized Likelihood Ratio or of Interactive Multiple Model-Based filters have been historically posed as solutions to certain event detection problems such as for failure detection in navigation systems and for maneuver detection in radar target tracking. This critical survey discusses specific drawbacks, barriers, and limitations encountered when attempts are made to apply these techniques in practical situations.**

***Index Terms*— Kalman Filter, Approximate Nonlinear Filter, IMM Filter, Radar Target Tracking, NAV Filter, Reduced-Order Filter, Failure and Maneuver Detection, GLR, Filter Stability.**

I. Introductory Overview and Summary Perspective

WE have an historical working perspective into several aspects of Kalman filtering [1]-[8]; including its generalizations to approximate nonlinear estimation [9]-[14]; and its related concerns [15]-[23] including having found, exposed, and corrected other historical fallacies [18]-[21], [24], even those relating to random or stochastic processes [25], [26] and in other Kalman filter related mathematics-based areas [22], [27]-[28].

We now seek to point out apparent weaknesses that have not been widely publicized or even acknowledged hithertofore that we, as specialist in this area, perceive to exist in several alternative approaches to *failure detection* (being a special case of *event detection* [29]). Such considerations arise in reducing mere theory to a final practical implementation instead of continuing to dwell on ideal starting points of the original formulation of an event detection approach without explicitly considering the realities of the constraints that exist in implementation within the actual applications. One such prevalent constraint being the standard use of reduced-order suboptimal filters [30]-[37, Secs. 6.7-6.9], with numerous application examples appearing in [39], where filter residuals are no longer ideally white and unbiased (specifically, filter residuals are white and unbiased if and only if the system and sensor model used in the Kalman filter are identical to what exists for the actual system or in it's *truth model* representation used in the simulation, otherwise the residuals are either nonwhite or biased or both [40]) thus degrading or corrupting the original idealized aspects of many detection approaches such as, for example, [41], which explicitly relies on an assumption of "whiteness and unbiasness of residuals" as a gauge of normal unfailed behavior. Please see the accompanying Fig. 1 which diagrammatically conveys in the parlance of a continuous-time representation the specifics being discussed herein using the familiar state-variable and Kalman filter notation popularized in [36] [1]. Although everything is actually implemented in discrete-time, the continuous-time version was used throughout Fig. 1 to make the ideas more straight forward. Although not depicted here as such, all matrices may be time-varying and the Gaussian white noises can be nonstationary as long as the associated covariance intensity levels and possibly time-varying means are completely specified. A more detailed but equivalent discrete-time version of just the system and linear Kalman Filter are available in [36, Figs. 4.2-2, -3]. Noises that are serially correlated in time may be routinely handled by appropriately augmenting the state to refect the Markov structure of the correlated noise as a response to white noise as the ultimate fundamental input stimulus [44], which makes the application situation again conform exactly to Fig. 1. Alternative approaches also exist within this framework for handling serially correlated Gauss-Markov noises (e.g., [45]-[47]).

The "whiteness of Kalman filter residuals" is also relied upon in another failure detection formulation using the *Generalized Likelihood Ratio* (GLR) [48], where, again, reduced-order filter usage introduces bias and nonwhiteness of the associated filter residuals even in the nominally unfailed situation. Such effects introduce ambiguity into the algorithmic decision of whether to declare that "a failure has occurred" or to declare that "no failure is present" since now the situation is less of a dichotomy for the decision algorithm after "the water has been muddied" by the use of a reduced-order filter, as historically required in most applications (where similar issues also arise for use of reduced-order observers [32], [33], [34] in application environments where noise is relatively less significant). Are the Kalman filter residuals now non-white and biased because of a failure occurring or because of the standard use of a reduced-order filter in the particular application? Such obscuring effects are consequentially time-varying when the associated navigation (NAV) filter structures which provoke or aggravate them are similarly time-varying (e.g., [50]).

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[1] This notation served as a standard for over 25 years in U.S. Department of Defense applications and elsewhere. Within the last 15 years, more recent books on Kalman filter technology (e.g., [42], [43]), unfortunately, no longer adhere to the prior standard, thus introducing a schism between the older and newer generation of practitioners over something as trivial as mere notation (that had been previously established by convention and concensus) which previously allowed easy cross-communications between researchers and practitioners when each knew exactly what the other was referring to. Of course, the benefits provided in the prior two references far outweigh the aforementioned drawback.

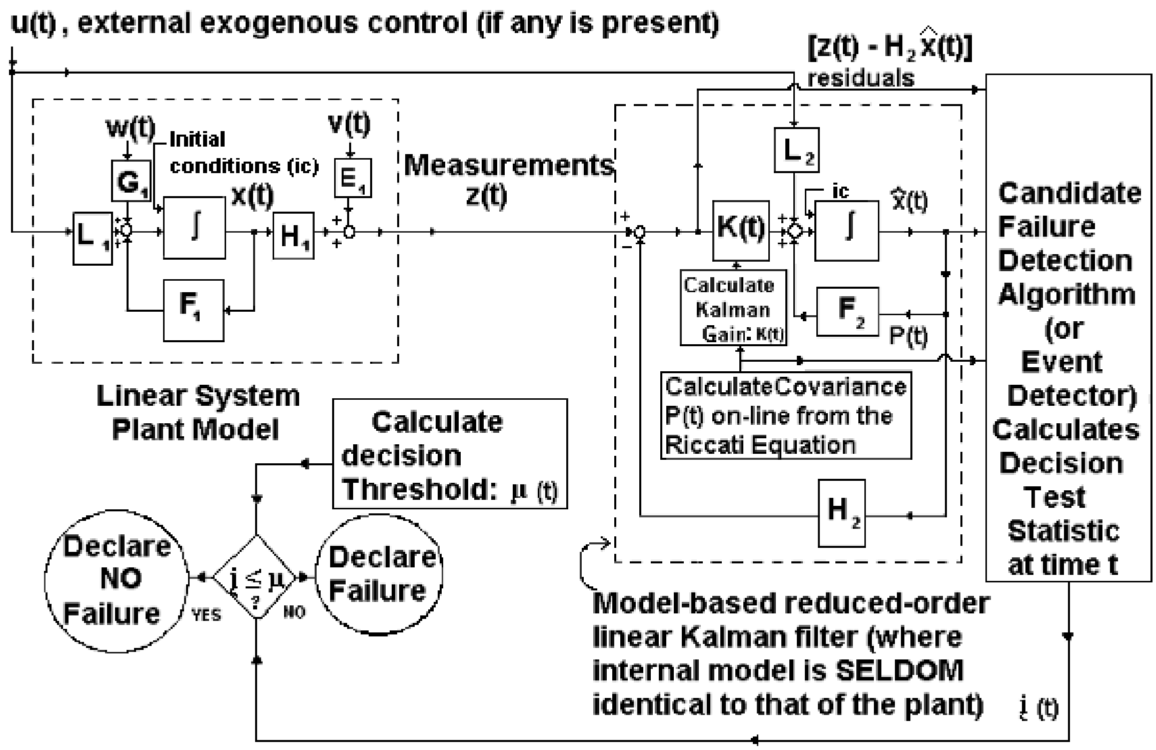

Fig. 1. Continuous-Time Summary of Navigation Failure Detection Algorithm's Structural Operations on Outputs of Linear System Plant Outfitted with a Linear Kalman Filter

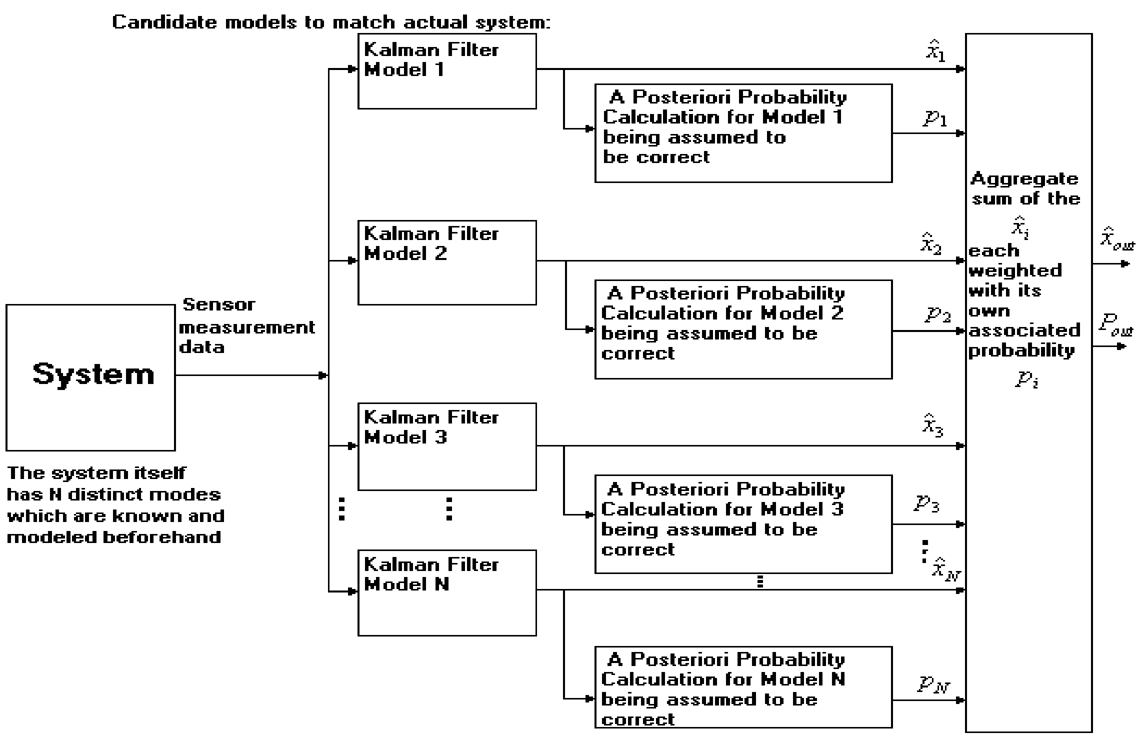

Fig. 2. Parallel bank-of-filters structure of Interactive Multiple Model (IMM) formulation for tracking systems that jump between a finite number, N, of alternative but specified modes (which have been modeled a priori)

While many issues above were raised in [49], [51], new approaches to failure detection have arisen that continue to exhibit the same weaknesses that had previously been warned of. Reduced-order filters can be made to perform adequately in applications by parameter tuning; however, the priorities in such tuning are:

1) preserving adequate Kalman filter computed output estimates of the states of interest;
2) secondarily, achieving adequate on-line Kalman filter computed output covariances.

The objective of item number 2 above is sought since these covariances may be needed within subsequent processing for failure detection algorithms for navigation systems, or within maneuver detecting algorithms in target-tracking radars, or within radar target tracking hand-over from detection radars to tracking radars, or in range gate overlap tests in making associations of new measurements with existing established targets within multitarget tracking methodologies [52], [53], which historically utilized some form of a solution to the *Assignment Problem* of Operations Research.

Both items (1) and (2) above are always the general goals being sought. However, in seeking to approximate an *ideal optimal estimator* using an Extended Kalman Filter (EKF) in nonlinear applications, the goal of item (1) is usually of higher priority than item (2) since associated EKF covariances computed on-line are suboptimal in general and are usually overly optimistic by being smaller than they should be (as computationally gauged by numerous Monte-Carlo evaluation trials to establish the true accuracy achieved). Alternatively, true accuracy can also be better ascertained for the case of nonlinear systems by using a Minimum Variance Least Squares Batch computing technique [54], [55] (which offers greater accuracy than offered by an Extended Kalman Filter but at the higher cost of processing all the available sensor measurements enmasse and so sacrificing being real-time in order to better gauge the accuracy of this algorithm's outputted estimates). Preserving "whiteness of the filter residuals" or *innovations* has not historically been on the list of any objectives in the design of adequately performing reduced-order filters (viz., [37, Secs. 6.8, 6.9], [38]), nor is this objective currently found within the cost functions associated with state-of-the-art filter tuning techniques now automating the plant noise covariance tuning process [56], [57], as an intermediate development step which seeks to robustify the Kalman filter that is ultimately implemented for the application at hand.

In navigation applications that involve a conventional Inertial Navigation System (INS) consisting of mechanical gyros and accelerometers, the adequacy of an approximating linear model is actively enforced by using actuators to periodically apply a torquing control to reset [58] the INS (causing it to be "stabilized") to help preserve the "small angle assumption" that is maintained as such for good linearity of the associated state variable INS error models [36], [37] (optical ring laser gyros, fiber optic gyros, whine glass acoustically vibrating gyros, and micro-machined gyros may be handled similarly after being stabilized). For such systems where linear models are adequate [2], outfitting with merely a linear Kalman filter suffices [37, Chapt. 6]. In contradistinction to the unfortunate situation portrayed in the above paragraph, Gaussian confidence regions still persist as ellipsoidal Gaussians even when reduced-order filters are inserted into an otherwise ideal formulation of the application [63]-[8] and certain reduced-order filters [68]-[70] still avail exact covariances on-line in real-time (as also confirmed in [4], [5], which also point out a down-side CPU loading aspect, confirmed by others [71]) so the Two Confidence Region approach is apparently robust with respect to this aspect when mechanized using these or other similar reduced-order filter formulations. As a specific example of how the state size of the actual NAV filter may be considerably smaller than that of the truth model [72]-[74], the truth models for SINS and ESGM were 34 and 100+ states, respectively, while the dimensions of the associated SINS-only, SINS/ESGM, and ESGM-only NAV filter models were historically (in the mid 1970's) 7, 15, and 18-states, respectively. However, the good news for event detection approaches based on the Gaussianess or on the associated Gaussian-based confidence regions [63]-[8] is that these tractable Gaussian confidence regions persist even when the underlying probability density functions (pdfs) are from the more general *exponential family* in the few situations where the important conditional and marginal distributions are still Gaussian [75, Chaps. 1-4]. Maximum Likelihood (ML) and Uniformly Minimum Variance Unbiased (UMVU) estimates for situations where additive noises from *exponential families* are present are both worked out in [76, pp. 284-294], [77, pp. 439-448].

II. A CONCISE HISTORICAL SUMMARY OF GLR DEVELOPMENTS

The Generalized Likelihood Ratio (GLR) approach to event detection, where maximum likelihood estimates of unknown parameters are utilized within the ratio of the H_1 pdf to the H_0 pdf in lieu of not knowing the actual requisite parameters of the mixed hypothesis (since they are in fact unknown), is presented and developed by Davenport and Root [78]. Root went further [79] to investigate applicability of GLR techniques in the radar detection problem of resolving closely spaced targets in a background of either known arbitrary correlated Gaussian noise or in Gaussian white noise. However, Root [79] obtained explicit criteria that could be applied to indicate conditions under which one could expect to <u>not</u> resolve two known signals (of unknown amplitudes and parameters) and additionally pointed out a difficulty of using GLR for this purpose.

[2]Within implementations of merely a linear Kalman filter in the feedback configuration for a Space Stable or for a Local Level Mechanized INS, where such techniques possess a nice linear error model system description, use of the INS-frame-to-computer-frame Ψ-angle misalignments are adequate. For any INS that is mechanized in a Strapdown configuration, instead of using a linear Kalman filter, the situation is sufficiently nonlinear and challenging to warrant use of an Extended Kalman filter as a real-time tractable approximation to an *optimal estimator* for such nonlinear systems. Only recently has the wisdom of this standard technical approach been questioned [59]. However, the jury is still out on this issue (see [60], [61]). Moreover, even better filter model representations for a strapdown mechanized INS have also been recently revealed [62].

McAulay and Denlinger [80] advocated use of GLR in conjunction with a Kalman filter in decision-directed adaptive control applications. Finally, Stuller [81] defined an M-ary GLR test that ostensibly overcame Root's original objections [79] to GLR for this type of application. (Ref. [81] also provides a limited history of GLR developments for radar, excepting no mention of [80], which may possibly have eluded him since it appeared relatively close to his publication date.) The use of GLR for failure detection was pioneered by Willsky and Jones [48] using an identical GLR formulation as presented by McAulay and Denlinger [80]. While both refs. [48] and [80] claim "optimality" of the GLR, neither explicitly specifies a *criteria* by which it may be judged optimal nor do they supply a proof or reference where such an *optimality* claim is demonstrated (specifically, [80, p. 231] references the English translation of [82] for "optimality" but diligent follow-up on our part here revealed no such substantiation located there).

While use of GLR has potential in many detection situations, it is not without its drawbacks that are frequently overlooked:

1) Selin found that some of the unknown parameters (such as unknown relative carrier phase) must also be estimated in order to maximize the *a posteriori* probability in the estimation of two similar signals in white Gaussian noise [83];
2) Selin further identified four standard caveats [84, p. 106] associated with use of a maximum likelihood estimate of the unknown parameters in a likelihood ratio (as occurs within GLR);
3) In general, GLR is not a Uniformly Most Powerful (UMP) test [85, p. 92], [49], [86, Exs. 2, 3, pp. 354-5], [87];
4) There are cases where use of GLR can give bad results [85, p. 96];
5) That use of a Maximum likelihood estimate (MLE) is not necessarily *statistically consistent* in general is explicitly demonstrated in a counterexample in [89, p. 146].

Within the last decade, GLR is again being advocated for use in radar (and also in active sonar) applications but those that advocate its use appear to ignore the historical objections raised against use of GLR in these types of applications as well as the explicit counterexamples in [49, 968 ff, App. A, pp. 973-974] that, apparently, have never been refuted. The new version of GLR (called "Ed Kelly's GLR") is of a different form than used by the others mentioned above [26] and is apparently a pseudo-GLR (since it ignores the time at which the signal event of interest was initiated, corresponding to the unknown location in time of a known signal [86, Ex. 2, p. 354]) but useful none-the-less for radar target tracking (as an approximate GLR). Also see its breakthrough use in new track-before-detect multi-target tracking [161] as a potentially revolutionary improvement. This new pseudo-GLR is less useful for failure detection situations since it ignores the underlying onset time of the detection event of interest. We speculate that use of the modern day Entropy Maximization (E-M) algorithm [91], [92] in conjunction with GLR to identify unknown parameters may, perhaps, now placate Selin's and Roots' concerns above and resolve item 5 above but E-M is a relatively large computational burden that may yet defy a real-time implementation and the details have yet to be worked out for this joint amalgamation. As summarized in [86, Introduction], there has been a flurry of activity in the last decade regarding the GLR test statistic itself (e.g., [88]), its properties, its structural invariance, and its generalization [90]; however, the other side of the coin of specifying the appropriate decision threshold to which the GLR test statistic must be compared in making correct detection decisions has received less attention (and its tractability is more challenging, as mentioned in [49, Ex. 2, p. 869; Exs. 4, 6, p. 970]). Decision threshold specification is necessary before adequate P_d vs. P_{fa} characteristics of GLR can be completely worked out to elucidate the associated Receiver Operating Characteristics (ROC) for use with GLR. A step in this direction is [93].

III. Loose Ends Apparently Plaguing IMM for Event Detection

We now turn to question the status of the Interactive Multiple Model filtering approach of Fig. 2 for practical systems exhibiting significant nonlinear dynamics in their associated plant models. While, by now, it is routine to consider the generalization of Kalman filter estimation techniques from mere linear systems (for which Kalman filters are optimal estimators [35]-[37], [76], [77]) to nonlinear systems (for which Extended Kalman filters or Iterated Extended Kalman filters [9] are frequently useful, tractable, approximate estimators for nonlinear filtering situations [16, Sec. 9]), as also discussed in [35]-[37, Vol. 2, 1982]. Similar ideas should successfully generalize each of the Kalman filters arising in the bank-of-Kalman filters that occur within IMM mechanizations as IMM is generalized beyond the exclusively linear case for which it was originally rigorously derived as a two level approximation (even in the purely linear case [49, Sec. 12]), where the *sojourn times* and *Markov chain transition probabilities* are new contrivances within the IMM Structure depicted in Fig. 2. These two new aspects are useful by providing additional parameters for tuning to better match potential application situations by keeping alternative models more actively viable than they had been for the original 1965 Magill bank-of-Kalman-filters [94].

A. Concerns about on-line IMM probability calculations

Unfortunately, the associated IMM probability calculations are more suspect in an attempted generalization to the nonlinear case. Specifically, in each of the following three references [95, after Eq. 2], [96, after Eq. 6], [97, before Eq. 4], "the critical mixture is *assumed* to be a sum of Gaussians, then the prior pdf is a Gaussian mixture and can be approximated (via moment matching) with a single Gaussian...." (While sums of Gaussian random variables or sums of Gaussian random processes are always Gaussian, that is not the issue or situation here where the topic instead is whether the resulting pdf of the output, as a weighted sum of the Gaussian pdf's called a "Gaussian mixture", does, in fact, coalesce into a single Gaussian, as claimed.)

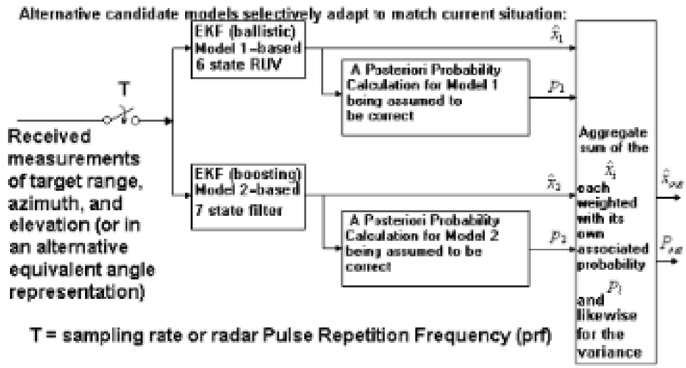

Fig. 3. A preliminary hypothesized IMM structure for tracking objects via radar as they transition between exoatmospheric and indoatmospheric regime or vice-versa (where the two filters have different state sizes)

Prior to the advent of [104], a statistical analyst could reasonably make the following four-fold objection to what had been asserted for IMM:

1) For nonlinear systems, the estimates outputted by an EKF are not Gaussian in general (unlike the assumption of Gaussianess that is invoked);
2) There are already existing analytic results [98] which caution that a single subsuming Gaussian pdf is not usually possible even if the individual participating pdf's were in fact Gaussian when the means of the various contributing pdf's are not in close enough proximity, as gauged by the spread of the associated covariances. This topic has been an issue since the historically well known Gaussian-Sums approach of [99], [100], which also used a bank-of-Kalman-filters structure (with an associated performance that also did not match "expectations", so to speak). Indeed, nonlinear filtering situations frequently exhibit multimodal output estimates as a fact of life, as discussed in [101];
3) The "moment matching" called for in [95], [96], [97] is not explained there (nor in their references) nor does an opportunity arise to perform such "matching" within the algorithm itself for each time-step k, as would evidently be needed;
4) It is not clarified what is to be matched in "moment matching" of what, and to what, and by what gauge will it be determined or established that it matches "closely enough".

Nothing about these four aspects had been explained in the three references cited above, where "moment matching" was called out as the requisite step to accomplishing the solution sought.

With the recent advent of [104] in its third paragraph from the end, its comparison of the probability calculations of IMM to those of the ideal, but unachievable, optimal filter makes the situation clearer but still somewhat unsettling none-theless. It demonstrates that these analysts specializing in this area can write down the requisite probability expressions that represent what is needed but cannot evaluate them conveniently (let alone in real-time when they are needed) because the problem is fundamentally infinite dimensional. The IMM probability calculations (which always assume linear models) are a tractable approximation that have no pretense of being close to what the probabilities should be when the underlying models are nonlinear, a situation that exists in some important applications, such as that depicted in Fig. 3.

The ideal expressions for the optimal estimator was provably optimal only if $M(k)$ in [104] were in fact known beforehand, then the augmented state $(x(k), M(k))$ would, in fact, be a Markov process as these authors rely on in their proofs. However, precisely how the system will switch is unknown to the analyst or application engineer a priori or even immediately after the evolution in time and transition in state of the system of interest which possesses this underlying multiple model system structure. This is yet another reason why their optimal system is unattainable beyond being infinite dimensional (corresponding to the need to solve an associated PDE [105]). The optimal expression for the probability calculation in [104, Eq. 21] is reminiscent of the Bucy representation for nonlinear filtering as in [106, Eq. 4], [107]. (Bucy's representation result is similarly found in [108, Eqs. 2.1, 2.2], but is a more tractable variation by utilizing a new intermediate result from [111] who used aspects associated with backwards Markov models.) Similar probabilities are evaluated in [104] and in Blom's predecessor publications for the assumed IMM application system structure. Other application evaluations of Bucy's representation are found in [108]. This early Bucy nonlinear filtering representation result, just mentioned, was firmed up and made more rigorous by T. E. Duncan [109], as beautifully and thoroughly explained in [110], [111]. (Because the Backwards Markov Model (BMM) technique plays such a prominent and crucial role in how Galdos obtained his nice result, it is desirable that present day researchers be extra careful in following-up on all chronologically listed insights and corrections to BMM [112]-[127] [3] that first arose just before and subsequent to the final results in [111] since errors were found and corrected in the "backwards Markov theory" at around the same time as Galdos' publication date and its prior review and subsequently continued to arrive in the installments reported above for another two decades. An old adage is that "imitation is the sincerest form of flattery." That definitely appears to be the case in this last sequence of references. Please notice reference [123] which claims a Backwards Markov Model precedent for A. N. Kolmogorov. This is most likely true since this same A. N. Kolmogorov [129] was also the first to recognize in 1933 that measure theory is the appropriate rigorous foundation for random variables and random processes (as acknowledged in A. Papoulis's textbook [128, footnote, p. 8] in 1965), and since engineering applications of random processes were avidly pursued in the USSR well

[3]Notice that in forwards-backwards Bryson-Frazier two-filter implementations of Kalman-like "optimal" smoothing algorithms using one filter processing forwards in time and the other filter subsequently processing backwards in time (see [113], [119], [127]), the Backwards Markov Model (BMM) theory plays a prominent role in determining how to convert the final conditions from the forward-time interval into the proper initial conditions to be used for the reverse-time segment over the same interval.

before the Western textbooks by the following well-known authors in the rigorous probability and random process area: J. L. Doob (1953) [130], M. Loeve (1963) [131], E. B. Dynkin (1960) [132], E. Parzen (1962) [133], A. Papoulis (1965) [128], R. L. Schwartz and B. Friedland (1965) [134], S. Karlin (1966) [135], W. M. Wonham (1970) [136]. For other similar Russian precedents, also compare early engineering applications of random variables and random processes in [137] vice those in [138], the latter having originally been published in the 1940's. Also compare the identical results in [139] vice [140, pp. 5-10].

B. Concerns about IMM Filter stability

For the hypothesized filter structure in the application of Fig. 3, the two different filter models need to be such that the state sizes are conformable. While conformability can be easily achieved by using a common state size of 7 throughout, for the exoatmospheric case this represents inclusion of a phantom state that does not really exist. Unfortunately, as experienced practitioners know from simulation experience, in the situation where the actual mode is in fact exoatmospheric (which really needs only 6 filter states to be present for Kalman filter tracking), some component of the received measurements will, unfortunately, be inadvertently attributed by the Kalman filtering algorithm itself to the superfluous phantom state thus depriving the true states of what should be attributed to them. The existence of [152]-[143] not-with-standing, this aspect is evidently still a problem (but apparently side-stepped in [144] at a cost of using 12-state filters for both situations, where previously six and seven state filters sufficed). Recall that the computational burden goes as the cube of the state size used within Kalman-like filters [145], [37, Sec. 7] such as these used in IMM so the implementation advocated in [144] incurs an additional CPU loading that is approximately $2^3 = 8$ times larger for processing and uses $2^2 = 4$ times as much for computer memory allocated than the alternative predecessor 6- and 7-state trackers together. Historically, the objective has been to seek the smallest state variable system model representation for use within the estimation algorithm. Indeed, the definition of a state variable representation is a first order vector differential equation that uses the fewest states (i.e., integrators) yet captures the essence of the physical problem. Further, besides [144] having a filter model that introduces more complexity into the exoatmospheric tracking application, the new risk is in using more states than are necessary and the associated likelihood of introducing *controllable* states that are not *observable*, or *observable* states that are not *controllable*, or states that are neither *observable* nor *controllable* [146]. Apparently, a necessary intermediate step needed to fully justify using a new model is missing since *observability* and *controllability* of the alternative 12-state target model that is introduced in [144] is not established there nor considered in predecessor research.

To remind the reader why this ancillary topic is so important, the *controllability* and *observability* aspect of the underlying mathematical model of the physical system (or comparably related *reachability* and *detectability*) play a prominent role in both classical filter tracker stability proofs [147], [149], [150, App. C] (as further explained in [2, App. B, pp. 3-35 to 3-36]) and in contemporary proofs for use with nonlinear systems [151], [152] of the stability of the Kalman-like filter in analytically guaranteeing that the tracking filter does an adequate job in closely following the true state of the system. Instead of utilizing the second method of Lyapunov, as the aforementioned rigorous proofs of Kalman filter stability do, [43] has no mention of Lyapunov functions or of Lyapunov's technique but, instead, the author argues that stability accrues by taking snapshots at each frozen instant in time, and for these, the filter/system error model has eigenvalues that are all strictly in the left half plane (for continuous-time) or within the unit circle (for discrete-time) thus indicating that stability is achieved even in the case of time-varying filter gains possessed by a Kalman filter. Unfortunately, this argument is fallacious when one considers the numerous counterexamples that have been historically unearthed [153], [155], one conveniently transparent example being the two state linear time-varying system of the following form [154, ex. 5.6, pp. 192-193]:

$$\dot{x}(t) = \begin{bmatrix} -1 + a\{\cos(t)\}^2 & 1 - a\sin(t)\cos(t) \\ -1 - a\sin(t)\cos(t) & -1 + a\{\sin(t)\}^2 \end{bmatrix} x(t) \tag{1}$$

with $a = 1.5$ and initial condition $x^T(0) = [5, 15]$. The solution to the above system may be easily demonstrated to be:

$$x(t) = \begin{bmatrix} e^{(a-1)t}\cos(t) & e^{-t}\sin(t) \\ -e^{(a-1)t}\sin(t) & e^{-t}\cos(t) \end{bmatrix} \begin{bmatrix} 5 \\ 15 \end{bmatrix} \tag{2}$$

by substituting this result and its time derivative back into the right and left hand sides, respectively, of Eq. 1 to yield an identity. Also, for $t = 0$, the specified initial condition is satisfied. Since the system is linear (although it is time-varying) with no singularities present and each element in the system matrix is bounded, the system matrix may be shown to satisfy a Lipschitz condition, therfore the solution depicted here is unique.

It may now be seen that for this system, in seeking for something like "eigenvalues" by a method reminiscent of the traditional route by first obtaining the characteristic equation:

$$\begin{aligned} 0 &= \det[\lambda I_{2x2} - F(t)] \\ &= \det \begin{bmatrix} (\lambda+1) - a\{\cos(t)\}^2 & -1 + a\sin(t)\cos(t) \\ +1 + a\sin(t)\cos(t) & (\lambda+1) - a\{\sin(t)\}^2 \end{bmatrix} \\ &= (\lambda+1)^2 - a(\lambda+1)(\{\cos(t)\}^2 + \{\sin(t)\}^2) + \\ &\quad a^2(\{\cos(t)\}^2\{\sin(t)\}^2) - (-1 + a^2\{\cos(t)\}^2\{\sin(t)\}^2) \\ &= (\lambda+1)^2 - a(\lambda+1) + 1 = \lambda^2 + 2\lambda + 1 - a\lambda - a + 1 \\ &= \lambda^2 + (2-a)\lambda + (2-a). \end{aligned} \tag{3}$$

The above quadratic equation may be explicitly solved for λ in closed form as

$$\lambda = \frac{(a-2) \stackrel{+}{-} \sqrt{a^2-4}}{2}. \tag{4}$$

Notice that both these constant "eigenvalues" are complex numbers having a negative real part for

$$-2 < a < 2, \tag{5}$$

yet the 1,1-term and 2,1-term of Eq. 2 is unstable for

$$1 < a < 2, \tag{6}$$

so, in particular, for $a = 1.5$, both the conditions of Eqs. 5 and 6 are satisfied and these so-called "eigenvalues" are constant and in the left half plane yet the solution of Eq. 2 is unbounded and unstable as time increases. This easily verified behavior contradicts some (widely propagated) notions by control theorists of an earlier era that believed that for time-varying "eigenvalues" (sic), if all confined to the left half plane and by not moving around "too much" [4], corresponded to a stable system. This example conforms with the first part of the desirata since its "eigenvalues" are constant and in the left hand plane (and the "eigenvalues" can not move around "too much" since they do not move at all) yet Eq. 2 is blatantly unstable and thus exposes the earlier notion as a "folk theorem" without substance. It is straight forward to establish filter stability using Lyapunov functions [156, Chaps. 3,4], [157], which avoid such unpleasant contradictions. Many other Lyapunov function successes in the analytic theory of various forms of Neural Network implementations are revealed and simply explained in [158]. Some 2^{nd} order time-varying linear systems can be handled using the rigorous Sturm-Louville theory [159] and eigenvalues and spectra are validly generalized in the Theory of Compact Operators [160, Sec. 9.8].

IV. Conclusion

The issues we raise above as likely drawbacks for failure event detection apparently also carry over for their mathematical dual in target tracking applications as well, especially when the underlying system models are nonlinear (as in space vehicle rendezvous, interception, or in reentry target tracking under inverse squared gravity models). These latter topics should be of high interest since they arise in National Missile Defense (NMD) and in Tactical Missile Defense (TMD).

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[4] Historically, the notion of time-varying eigenvalues moving around "too much" was not precisely elucidated and, as such, was sufficiently loose by allowing enough wiggle room to accommodate any situation. Therefore, this particular aspect was the loophole in the above "folk theorem" that allowed those historical experts to escape from their earlier pronouncements without questioning the validity of the theorem. Having constant "eigenvalues" in this numerical counterexample makes the noose tight enough so that there is no wiggling out.

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