Novel Variations on Old Architectures/Mechanizations for New Miniature Autonomous Systems

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Abstract

Options for handling an INS are explored, as motivated by how things had been done on U.S. submarines.

We at **TeK Associates** have investigated cutting edge navigation concepts since the 1970's [1]-[2], including early use of GPS as well as its use at high update rates to compensate for the higher drift-rates associated with use of a less expensive INS (e.g., under sub-contract, used Kalman filter-related covariance analysis to perform quantitative analysis of the relative pointing accuracy provided by each of several alternative candidate INS platforms of varying quality and cost by using high quality GPS [P(Y)-code, differential, or kinematic fixes] at a high rate to enhance the INS with frequent updates to compensate for degradations incurred with time due to specific inherent gyro drift rates characterizing each INS, as representatively conveyed in 5 figures at the end: one for each INS, a summary, along with one representative intermediate simulation depicting the characteristic sawtooth before and after each GPS fix). We are cognizant of operational principles of GPS [26]; of Inertial Navigation Systems (INS) [6], [33], [48], [77] and their underlying linearized error models [6]; of Kalman filter estimation theory [22], [24], [29]-[31]; of reduced-order filters [23], of host platform [27] and environmental constraints and countermeasures [21], [28] for theoretical analysis, simulation, and software implementation:

- Trail-blazed development of a particular Kalman filter accouterment: **Two C**onfidence **R**egion (**CR2**) Failure Detection, from first principles by developing a test statistic and subsequently specifying False Alarm and Correct Detection Probabilities that are traded-off in specifying **CFAR** time-varying decision threshold for comparing it to in making fail/no-fail decisions [10]-[19].
- Surveyed and analyzed alternative **Decentralized Filtering approaches** to identify those satisfying constraints possessed by NAV applications of interest to NADC (for JTIDS RelNav) [8] and to Wright-Patterson AFB for **MUFBARS** and **ICNIA** for Advanced Tactical Fighter [25], first under **ITT** contract then for **TRW**.
- First to recognize the utility of combining earlier "failure detection methodology" with that of "decentralized estimation", thus reaping a satisfying firm theoretical foundation for **"redundancy management" for airborne navigation applications** consisting of an INS with position fix/resets from several alternative external navaids for updates [9].
- Trail-blazed analysis and evaluation of multi-sensor data collection using INS/GPS to support a particular **airborne platform and its mission of collecting terrain data** [6].
- Published a **critique of existing GPS** [20], [21].

We are familiar with problems associated with many emerging Unmanned Autonomous Vehicles that need remedying:

1. **Lack of redundant gyros (and accelerometers)** in using only three orthogonal single-degree-of-freedom conventional mechanical spinning rotor gyros so UAV design is not robust with respect to incurring even a single routine gyro failure (or accelerometer failure) which can then jeopardize the success of its mission. (See [8], [9]-[19], and alternate approaches to GPS RAIM for some answers.)
2. **Lack of any coherent calibration procedure** to get the inertial navigation system up and operating well (i.e., accurately) after having been stored on a shelf for awhile. (See Appendix A for an overview answer.)

Use of Rockwell Micron INS, which consists of electromagnetically supported spherical gyros, possess two input axes per gyro and, as such, is known as being a two-degree-of-freedom gyro and, as a consequence, having just two gyros provides a redundant input axis. So use of, say, three Micron gyros can then provide full redundancy (when 2 input axes from different gyros point in each of the 3 directions) [41]-[46]. **R**ing **L**aser **G**yros (RLG) possess excellent shelf-life characteristics that tend to exhibit the same constant random white noise level and (random) constant bias trends, as originally established computationally during initial calibration of the system, as performed, perhaps, years earlier. Unfortunately, the accuracy of a RLG goes directly with the area circumscribed by the perimeter of its laser light path so a small RLG has low accuracy and vice versa.

The typical mechanical spinning-rotor gyroscope found within classical conventional Inertial Navigation Systems (INS) is constructed by suspending a relatively massive spinning rotor inside three orthogonally mounted support rings called "gimbals". Mounting each of these rotors on axes with high quality bearing

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surfaces with low friction insures that very little resisting torque is exerted on the inside rotor as the rotor continues to spin. Electrical “torquers” are utilized to initially spin up the innermost rotor to its nominal speed and to maintain its angular velocity, as continuously monitored by electrical or mechanical “pick-offs”. Modern day INS gyros take many different forms [33], [48] based on whatever particular inherent physical principle is being exploited in a particular device, such as that for wine glass acoustic frequency vibrating gyros, for electro-statically supported spherical gyros, for electro-magnetically supported spherical gyros, for ring laser gyros (typically providing a modicum of dither to avoid laser lock between the two opposing paths), for fiber optic gyros, for atomic quantum spin gyros, etc.

Another important aspect is just how two (or more) gyros (and their associated accelerometers) are ultimately implemented or bound together within an Inertial Navigation System (INS) complex such as in a Space Stable configuration, in a Local-Level configuration (as either Wander Azimuth, Free Inertial, North Slaved, or North Pointing), or in a Strap-Down configuration; and in its corresponding associated Navigation filter (i.e., Kalman filter) formulations, which can be implemented in three different alternative ways [76] but with the differential feedback form being prevalent nowadays. New insights have been recently revealed in Ref. [3] into how modern MEMS gyros may now be implemented in Strapdown configurations without the same hassles or operational constraints being imposed that were historically associated with handling the classical spinning rotor gyros discussed above.

A early 1970's era Draper Laboratory study [78] had concluded that a preferred optimal configuration for redundant gyros (with only one-degree-of-freedom input axes) was being located and oriented in a certain prescribed way as an orthogonal triad placed along each of the 12 faces of a regular dodecahedron. Perhaps as Draper Laboratory now wrestles with the cheap and plentiful MEMS gyros (that have a coarse accuracy), Draper should consider again invoking their own historical 40-year-old solution (but they have not done so yet). See properties of a regular dodecahedron at http://en.wikipedia.org/wiki/Dodecahedron. Its faces are pentagons.

Reference [3] contains a good modern perspective. What is conveyed in Ref. [3] is possibly of revolutionary importance! However, the assertions and conclusions in this paper should be viewed as being very controversial and examined closely (as eventually occurred in greater detail in [65]-[69]), where conclusions are somewhat more conservative and less adamant in these subsequent studies). Also see Refs. [4], [5], [34], [36]-[39], [41] for other perspectives. Strapdown is indeed the dominant INS mechanization for all airborne applications these days as has been the case for the last 25 years. However, Strapdown, by bolting the gyros and accelerometers directly to the body frame of a particular platform and accounting for platform motion relative to these INS sensors, is a little different and more computationally intensive than the other available INS mechanizations (all being nonlinear). The challenges and issues associated with Strapdown have been addressed in the open literature of navigation as the solutions evolved. Yet the many alternative mechanizations for an INS, already listed above, and ways to use Gimbals that were once the weak link (since four nested gimbals were needed to avoid incurring gimbal lock), and, in the past, there was significant friction to be encountered and overcome in their use, and mechanical torquers were needed too. Technology for all of the above has greatly improved by now and may be worth another look.

Other Options to maintaining INS accuracy (Spawned by How Navigation had been handled on Submarines)

If we temporarily dispense with or suspend the requirement that an airborne INS may only be implemented in Strapdown only, then the following techniques can be utilized:

- **Gyro Monitoring** for a Local Level mechanization of three single-degree-of-freedom gyros, each mounted orthogonally to the others, involves using a fourth independent but similarly constructed gyro, mounted in the same plane as two of the input axes of the other gyros participating in the computed navigation solution and then periodically rotating the so-called extra monitor gyro to align itself with first one and then the other of the two "horizontal gyros" of the INS complex with their input axes in the same plane to obtain the sum and difference of the respective constant bias gyro drift-rates. The gyro biases of the two gyros whose input axes are in the horizontal plane can then be computationally estimated by solving the two appropriate linear equations in two unknowns [32], [42]. A psi-angle analysis yields the appropriate linear equations to be used for this, where the psi angles are the standard gyro-frame to computer-frame misalignments (described by linear INS model ODE, driven by the drift rate noises and biases, and scale factor errors).

- The **HAD/HAP** procedure (of the late William Zimmerman) could be invoked again to measure and compensate for the effects of biases by also how it constructively used redundant navigation elements.
- Principles incorporated into the **Carousel INS** [52] (patented by William Zimmerman, Robert Shipp, et al in the 1960's) can be invoked. The entire complex is rotated at a constant rate effectively averaging out the effect of constant bias drift-rates present in the single-degree-of-freedom gyros within the plane of rotation.

The first two of the above three techniques were developed for submarine navigation, where the submarine "flies" through the water like an aircraft flies through the air, as its fluid. A single monitor gyro would need to be located on each of the 12 faces of the dodecahedron upon which a triad of gyros and accelerometers are mounted. Each monitor gyro would need to be rotated to align correctly with the other gyro's input axes along the particular dodecahedron face of concern. Similar steps could be taken to implement a Carousel-like INS complex or **HAD/HAP** mechanizations for each triply orthogonal pair of input axes occurring on each face of the dodecahedron. Moreover, for logistics and re-supply, it is not out of the question to have appropriately hinged faces of the dodecahedron made of stiff plastic or some sort of polymer so that it packs flat and, perhaps, benefits from this unfolding during initial calibration of all the participating (modestly priced systems at $60) **MEMS** gyros [47]. However, a solid dodecahedron would suppress vibration and flexure. Or unfolded could later wrap around a solid core. Ultimately, the final NAV solution of primary reliance could be the average of the 12 participating independently computed separate NAV solutions offered on each face of the dodecahedron (each with covariance sigma squared); and, as a consequence, the final average would have an accuracy improvement consisting of sigma squared divided by 12 (std. dev. = sigma divided by square root of 12). This same technique can also be used to improve the accuracy of RLG's through averaging to yield one consensus output for primary reliance. There is still a need to damp the "vertical channels" with either altimeters or GPS [35], [40], [70], [71]. While augmentation with an altimeter was at first proposed for use to improve GPS availability when less than 4 GPS satellites were available to compute the four quantities of 3 geographical coordinates and time, new results avail GPS position with far fewer satellites [72], [73] and can now be used to cross-check the altimeter. See caution about possible excessive reliance on GPS following Table 1 in App. B.

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Appendix A: INS Calibration/Alignment and Distinctions Between Strapdown and its Alternatives

Mathematically, Inertial Navigation Systems (INS) alignment is the process of expediently determining appropriate initial conditions for the integration of the underlying differential equations describing inertial navigation [49][50]. Six coupled second order differential equations for the attitude and position of the host platform are involved, thus establishing the need to specify 12 initial conditions as, nominally, being 3 position, 3 velocity, 3 "platform" orientation angles, and 3 "platform" rate coordinates.

Since the INS (in the normal mode) is inherently a *dead-reckoner*, which integrates open-loop measurements of acceleration into, first velocity, then into position; it is incapable of fully self-contained initialization (within a short enough time frame to be practical) so that the INS must usually depend on inferring the necessary information for initialization as supplied from an externally sensed position (and/or velocity) reference. The INS alignment process is thus normally defined as the estimation and/or control of "platform" attitude and its rates through the observation of their effects as errors in the measured position and/or velocity states.

This description of the essence of the alignment process is universally applicable to all of the available alternative INS (nonlinear) mechanizations listed above, the sole distinction being how gyros in a Local-Level mechanization are torqued to bring the platforms into physical alignment with the desired frame, versus the computational "torquing" involved in establishing the rotation representation in Strapdown systems. Strapdown system alignment is actually simpler, since it avoids complications otherwise introduced in implementing conventional gyro torquers with their nonlinear constraints (or stops) on the practical limits that can be achieved by the torquer in seeking the goal of perfectly leveling the stable platform (see [51] for alignment details in handling a strapdown laser gyro INS as well as [38], [41], [52]-[55]).

The concept of alignment basically reduces to efficiently and expediently tracking a pair of known reference vectors, which can be directly sensed by the inertial instruments and which contain enough information to suffice in completely defining a reference frame. The obvious choice to be reference vectors for terrestrial navigators are the gravity vector (for "leveling") and either the earth angular vector (for earth-rate gyrocompassing for ground-based alignment) or the vehicle angular velocity vector with respect to inertial space (for "space-rate gyrocompassing" for quick reaction but, in general, coarser in-air alignment). Both of these approaches encounter singularities if the two vectors selected become collinear or if one of these vectors vanishes (due to inadvertent sensor failure). An additional option for performing an in-air alignment [43], [56] is to create externally trackable (by a cooperating external tracking radar) specific force vector(s) by executing dynamic maneuvers in-flight to provide information that is feedback to the aircraft via a communications link.

There is an apparent advantage of potentially reducing the time required for an alignment by using dynamic alignment since an accelerometer stimulus operating through "platform" attitude error produces a more immediate effect (i.e., is one integration less removed) in its effect in the observable states than an angular velocity stimulus would be. However, the mechanism is nonlinear and may take longer to converge to the answer. A final aspect of the alignment process involves the "transfer alignment" from a Primary Master INS to a Secondary Slave INS. If the Primary INS system data is utilized solely as a position/velocity reference for the Secondary INS, then the process constitutes a so-called *standard alignment*. However, more rapid response can be obtained in this final alignment phase by exploiting the analogous inertial nature in common to both Primary and Secondary INS as expedited by employing either accelerometer matching (of sensed gravity) or attitude matching (being the two most common examples of using this technique). For alignment on aircraft carriers, velocity matching is typically used between Primary shipboard INS and the Secondary aircraft INS, while on the flight deck (and similarly for aligning missile gyros from the shipboard INS on submarines). Non-ideal alignment disturbances (which interfere with the normal alignment process) are usually also present as unknown Master INS errors, local vibrations, and uncompensated bending/flexure between the locations of the Primary and Secondary INS.

INS alignment, by the slaving of observable states to an external reference, is well suited for Kalman filter-based optimal estimation [49], [57]-[60] unless the initial estimates are not within the region of close linear approximation. If this happens, convergence time may suffer unless one uses an Extended Kalman Filter [43], [61] or parameter identification techniques [62] in a practical "coarse alignment phase" prior to the "fine alignment phase". One coarse alignment procedure is to use the accelerometer outputs in conjunction with external measurements of aircraft velocity to directly torque the gyros (or the rotational representation in strapdown mechanizations). After the initial transients have died out, the difference between the indicated and measured aircraft velocities are proportional to the components of earth rate along the level axes and can be utilized to derive an estimate of the initial alignment of the "platform" frame relative to North (assuming that excessive aircraft sideslip is not being experienced). Normally, Local Level systems will cage to aircraft axes on power-up, while strapdown algorithms can be initialized a priori under the assumption of zero pitch and roll.

The fine alignment phase usually utilizes Kalman filter estimation of residual attitude errors by processing the available external observations. The filter settling time for this is about $1/4^{th}$ a Schuler period, but is an order of magnitude faster (by about 2-3 minutes) if dynamic alignment is utilized by having the aircraft perform horizontal planar maneuvers (S-shaped turns) to enhance *observability* aspects discussed below, where *observability* is a notion associated with use Kalman filter technology and is fundamental to its successful use.

A known difficulty with gyro-compassing procedures [63, pp. 1161-1163] is the non-separability of azimuth error from effective East gyro drift-rate and the non-separability of "platform" tilts from effective East and North accelerometer biases under conditions of quasi-constant alignment of instrument and geographic axes. *Observability* resolution enhancement techniques [53], [54], [42], [44]-[46] involve changing the relative orientation of the axes (through aircraft motion for strapdown mechanizations or by prescribed azimuth slews for Local-Level mechanizations).

Appendix B: Model Parameters used in Obtaining Numerical Evaluations (Gyro & Accelerometer Biases, Scale Factors, GWN rms, Markov Time Constants from Each Manufacturers' INS specifications)
Assuming in the Kalman Filter-Based covariance analysis calculations that follow:

1. SEP 3m, fix rate 1 Hz (differential solution),
2. SEP 0.05m, fix rate 10 Hz (kinematic solution).

Using [75, Table 5.3, page 75] to convert 0.50SEP = 3 meters to standard deviation (i.e., $1.5382 = k = R/\sigma \Rightarrow \sigma = R/1.5382 = 3$ meters/1.5382 = 1.95033 meters $\Rightarrow \sigma^2 = 3.80379$ meters^2). Assuming that it is available in stationary (i.e., non-moving situations when enough time is allotted to obtain a good average of GPS solutions, as occurs in surveying applications) similarly used [75] to convert 0.50SEP = 0.05 meters to standard deviation (i.e., $1.5382 = k = R/\sigma \Rightarrow \sigma = R/1.5382 = 0.05$ meters/1.5382 = 0.032506 meters $\Rightarrow \sigma^2 = 0.001057$ meters^2). Similarly, the 0.95CEP = 9 meters is converted, via [75, Table 4.3, p. 47], to $2.4477 = k = R/\sigma \Rightarrow \sigma = R/2.4477 = 9$ meters/2.4477 = 3.676921 meters $\Rightarrow \sigma^2 = 13.519749$ meters^2. Ref. [74] provides a description of the μFOR S-1 FOG along with a description of the Rockwell Collins GPS NavStrike dual frequency, 12-channel (all in view), and the SAASM-based 3.00" by 3.5" Precise Positioning Service-Security Module used to store and process the Crypto key. Claim in [74] is that this configuration is currently used in the HARM missile (as of 2002).

According to [74], the distribution of crypto classified red keys to hundreds or thousands of users worldwide is now recognized (in 2002) as a logistical nightmare. The U.S. Government developed a requirement for new security architecture, SAASM, as support for Over-the-air Re-keying (OTAR). SAASM is intended to replace the current PPS-SM in GPS receivers. The U. S. Government has issued a mandate that all new GPS programs must have incorporated SAASM by fiscal year 2003. The code search of this NavStrike GPS receiver is 25 times faster than it was for GEM III. The Key Data Processor (KDP) performs all security algorithms used to compensate for the effects of Selective

Availability and Anti-Spoofing. Ref. [74] acknowledges that SA is supposed to be going away but that this capability is still required for these DoD applications.

Ref. [74] states that NavStrike GPS accuracy is 17 meters (95% SEP) in 3-D; 9 meters horizontal (95% CEP) in 2-D using P(Y)-code and code and carrier tracking. This is quoted using GPS-GRAM-001A model. Ref. [74] claims actual results will be even better because Space Segment and Control Segment Errors are better than spec and that there are currently (in 2002) more satellites than originally anticipated.

In our flight path (12000m diameter circle) the only angular accelerometer getting drift-rate help from the GPS updates is the azimuth (heading sensor). (This is aside from any reset levels offered from a well-modeled g-vector). We are moving at a rate of about ((60-m/s/12000pi-m) * 360-deg) 0.57-deg/s, large compared to the velocity induced uncertainty of 2SEP/s ~06-deg/sec. Assuming the Kalman filter to reset the error drift rates to this level or better at each epoch, it appears that in our flight path, so long as the delta-Vs are un-resolved (below 2SEP/s), the GPS update does not provide a reset any better than this drift. In our simplified interpretation, we are resolving a least squares determination of a single point (which we already know as being 2SEP) instead of a time varying function. This means that we would have to induce a non-zero input in the form of an aircraft pitch or roll that would be observable in the psi-angles. Our pointing accuracy requirements and the drift rate of the sensors drives how often this is needed. This being said, it appears that it is the error drift rate, not the error, that is constructively limited or ameliorated by the GPS updates and we would therefore be better served to update GPS at a different (considerably longer) epoch from the linear accelerometer requirement. If, say, we used a 10 second epoch, we would hold IMU azimuth drifts to less than 0.006deg/s for differential operations

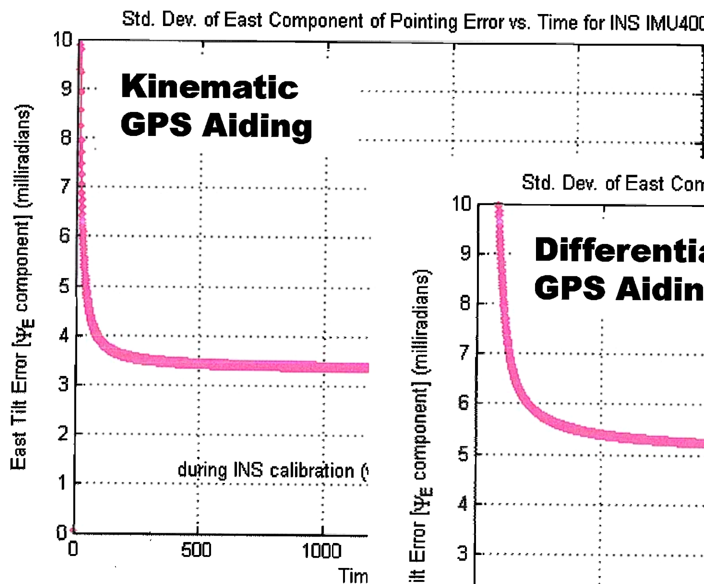
Table 1: IMU Sensor Comparison Chart [provided by William Morris **(Arête)]**

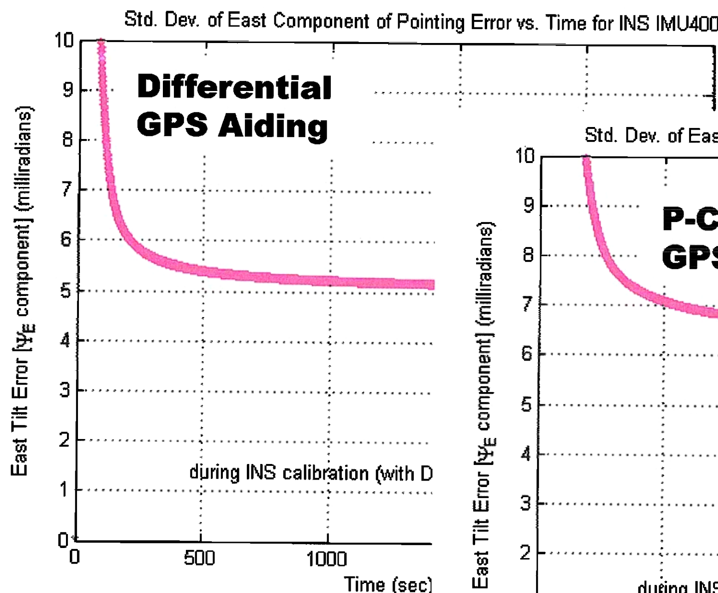
Parameter	LN200	IMU700	C-Migits/DQI	IMU400	MotionPak	MotionPakII
Angular Rate						
Range	1000deg/s	200deg/s	1000deg/s	200deg/s	>100deg/s	75deg/s
Bandwidth	>500Hz	100Hz	100Hz	25Hz	60Hz	30Hz
Bias Deg/rt.Hz	0.04	-	-	-	-	-
Deg/s	-	0.03	-	2	5	5
Deg/hr	1	20	10	-	-	-
Variation	0.35deg/Hr	-	3deg/hr	-	-	-
Stability	-	-	-	-	2deg/s/yr	-
Scale Factor	0.01%	2%	0.035%	1%	6%	6%
Random Walk D/rt.hr	0.04	0.4	0.035	2.25	-	-
White Noise	-	-	-	-	0.01deg/s/rtHz	0.2deg/s 0-30Hz
Non-linearity	-	1%FS	-	0.3%FS	.05%FS	3%FS
Non-orthogonality	0.1mrad	0.2mrad	-	-	-	-
Acceleration						
Range	40g	10g	20g	10g	10g	3g
Bandwidth	100Hz	75Hz	100Hz	75Hz	300Hz	250Hz
Bias	+/- 0.2 mg	+/- 12 mg	+/- 1.5 mg	+/- 12 mg	+/- 12.5 mg	+/- 200 mg
Scale Factor	.03%	1%	.035%	1%	-	5%
Non-linearity	-	1%FS	-	1%FS	.05%FS	3%
White Noise	.05mg/rt.hz	-	.06mg/rt.hz	-	10 mg rms	3 mg rms
Random Walk	-	0.5 m/s/rt.hr	-	0.5 m/s/rt.hr	-	-
Non-orthogonality	0.1mrad	0.2mrad	-	-	-	-
IMU Weight (lbs)	1.5	3.5	2.2	1.4	2	2.8
IMU Price (2003)	~$40k+	$11.5k	~$25k/$18k	$4k	~$12k	$3k

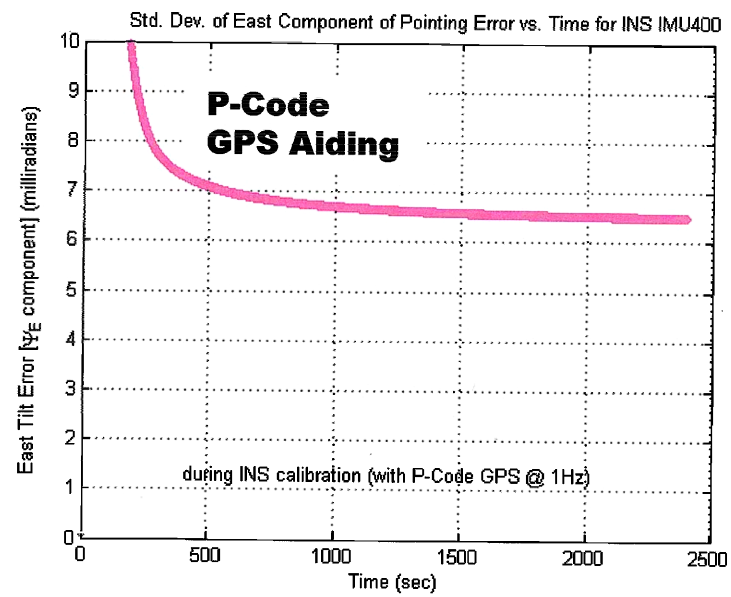
Navigation covariance analysis was performed under contract by TeK Associates using INS parameters from columns 2-5 to obtain results.

TeK Associates is concerned that a dangerous trend has been allowed to persist by U.S. Defense systems myopically dwelling on using the cheapest INS to do the job wholly dependent on being augmented by frequent GPS fixes. Such reliance is not realistic in critical wartime situations where serious jamming, perhaps from escorts, would be present and the GPS satellite broadcast powers from 11,000 nmi away could be relatively easily swamped [21]. This aspect is aggravated by the demise of Loran-C after 2009 that could have crosschecked GPS.

Single Axis Pointing Accuracy for IMU400

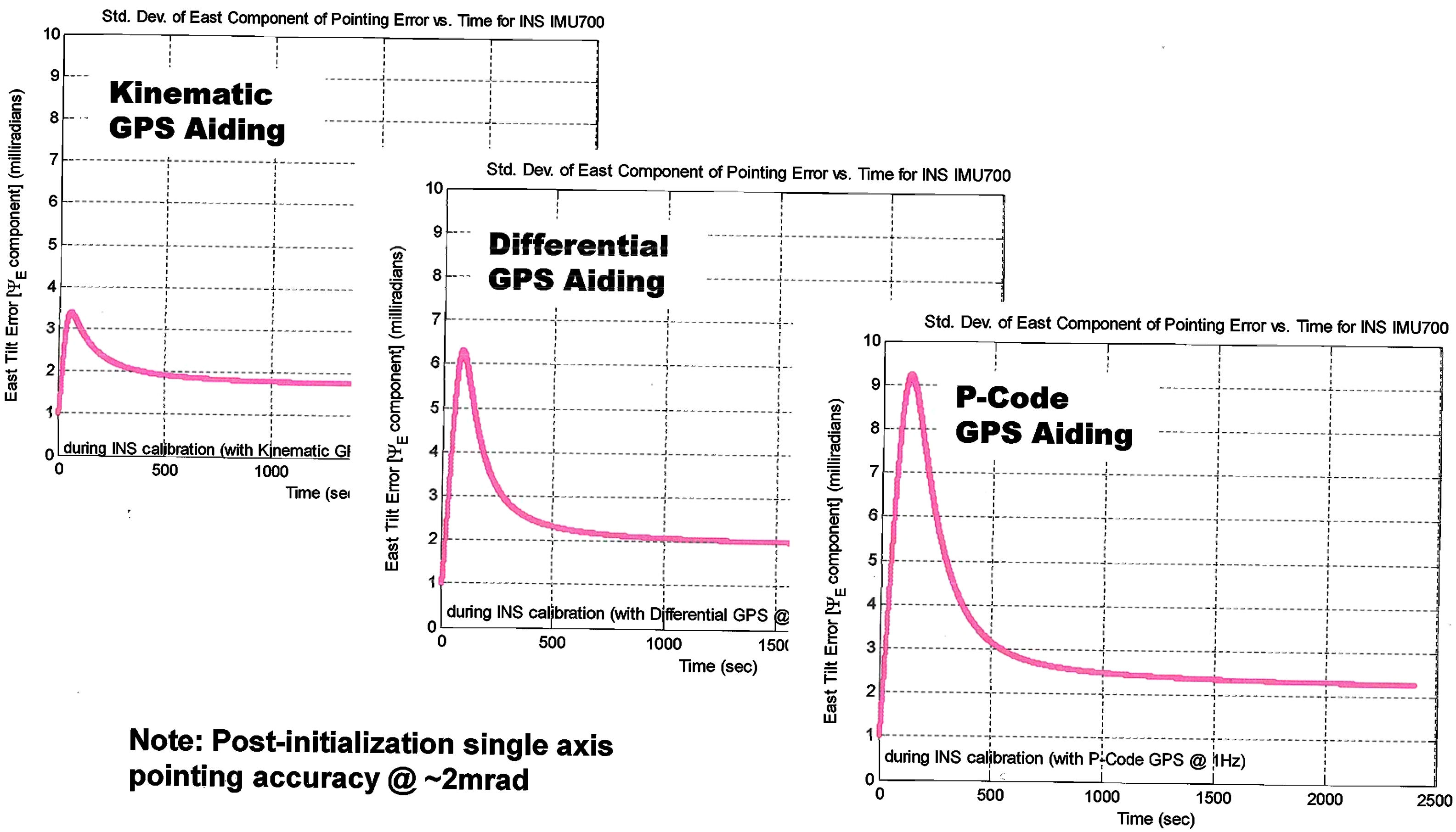






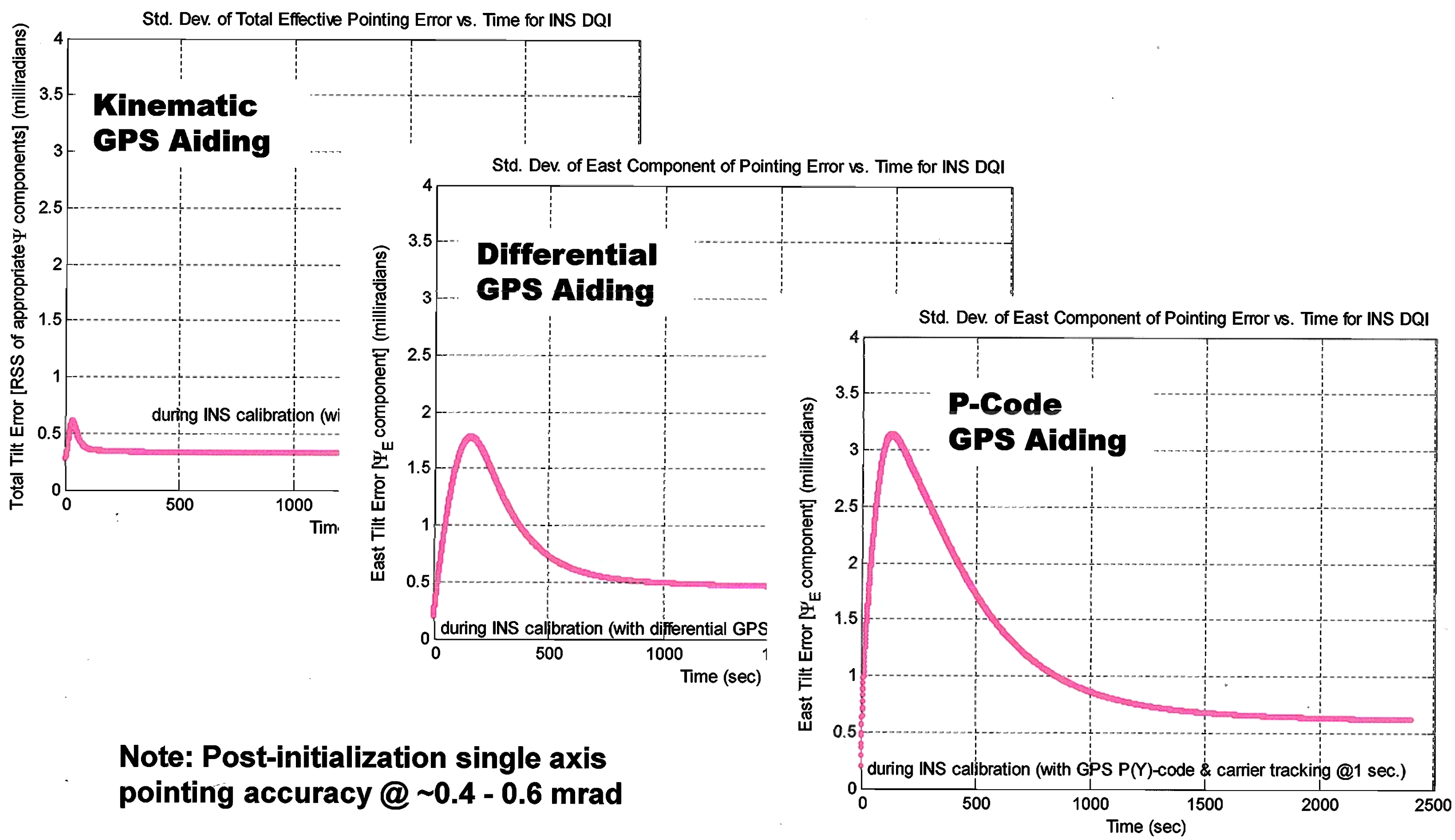
Note: Post-initialization single axis pointing accuracy @ ~3.5 - 6.5 mrad

Single Axis Pointing Accuracy for IMU700



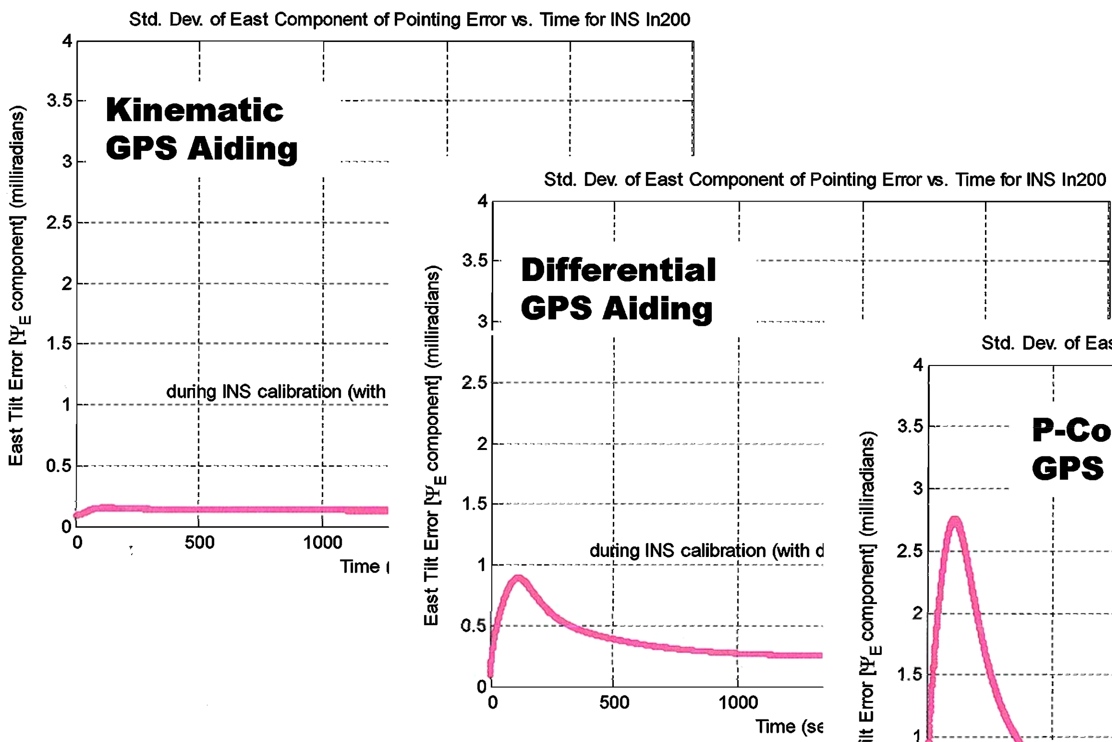
Note: Post-initialization single axis pointing accuracy @ ~2mrad

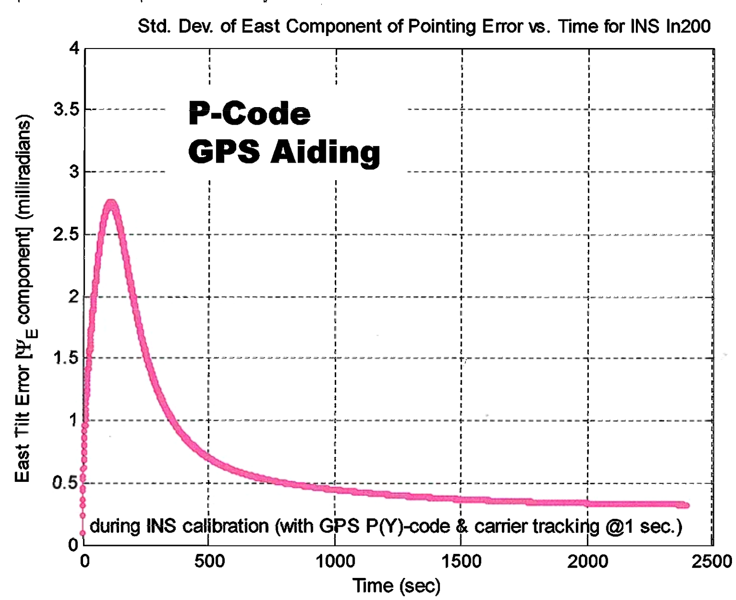
Single Axis Pointing Accuracy for DQI



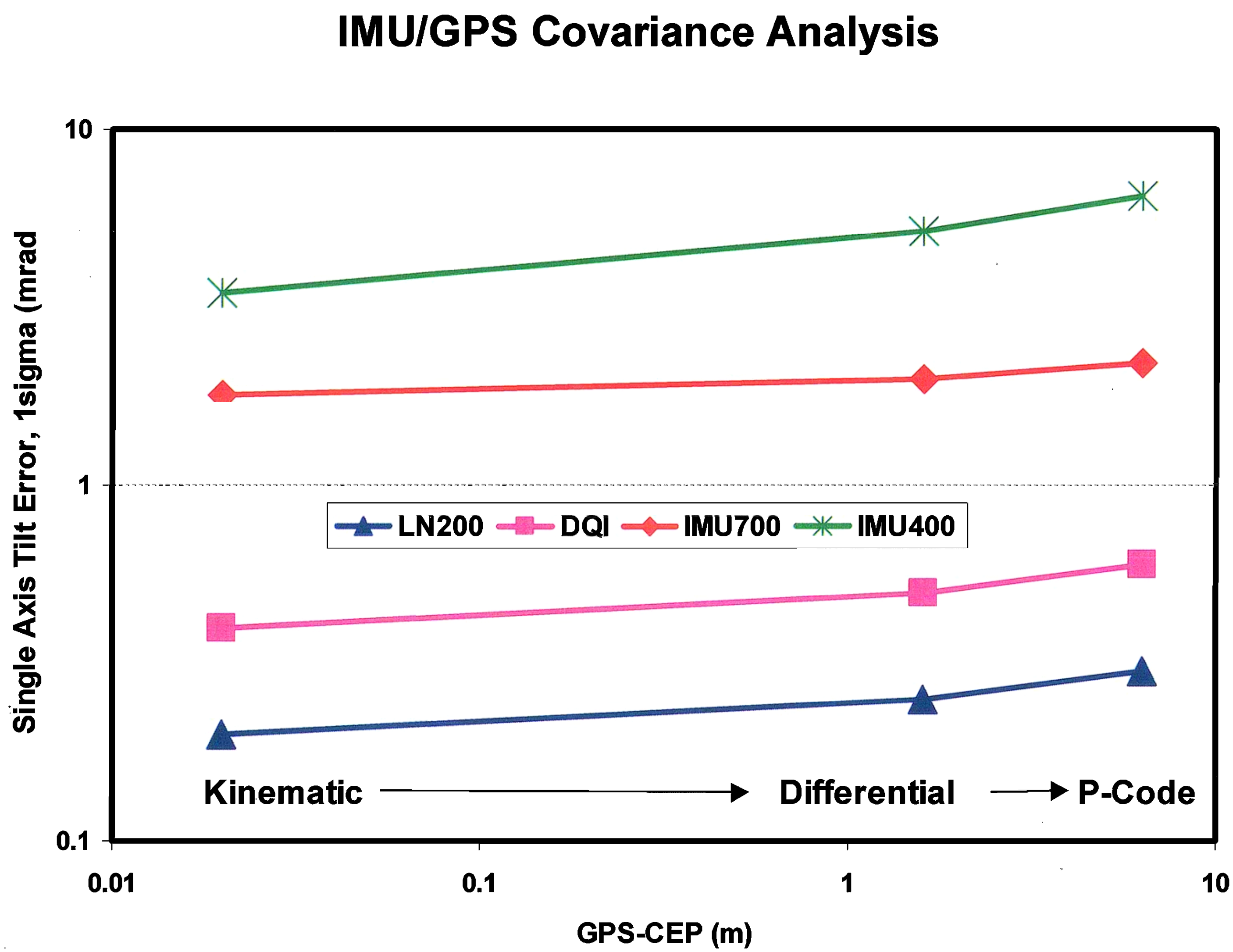
Note: Post-initialization single axis pointing accuracy @ ~0.4 - 0.6 mrad

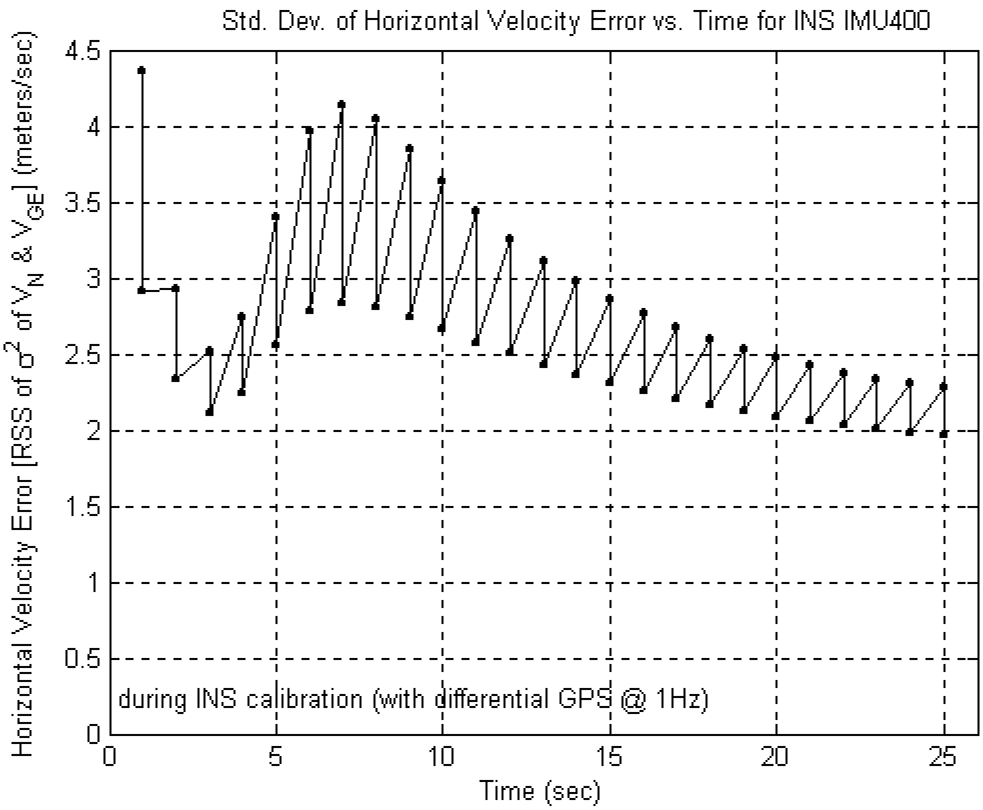
Single Axis Pointing Accuracy for LN200

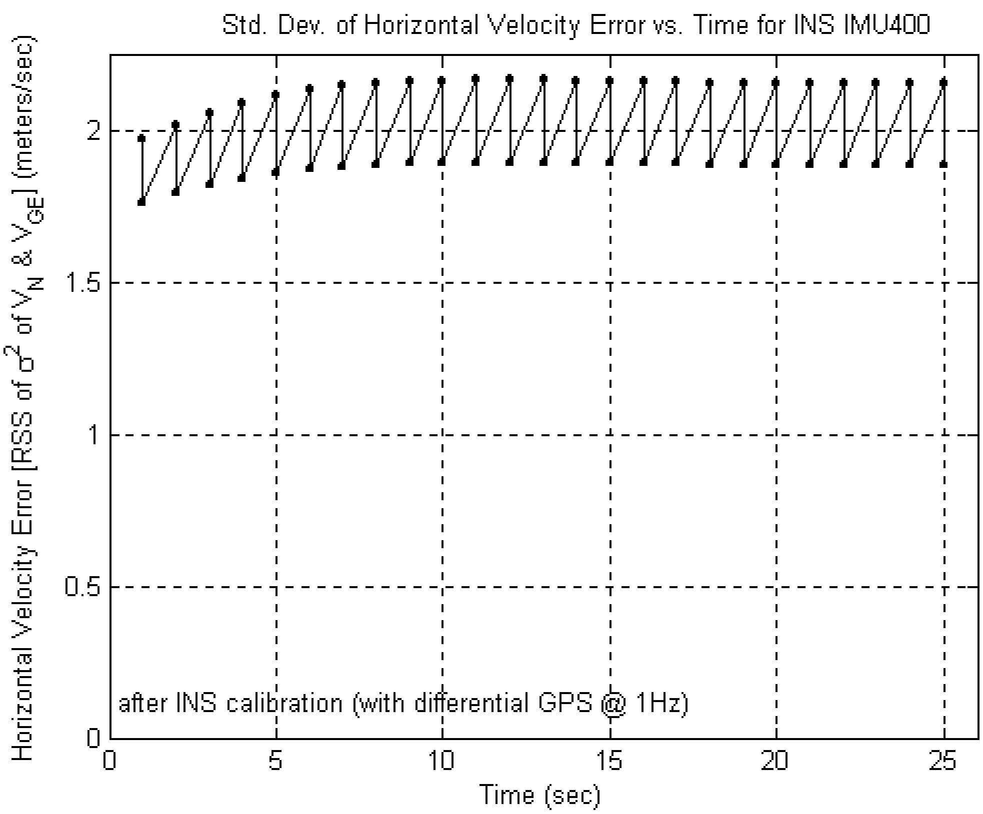




Note: Post-initialization single axis pointing accuracy @ ~0.2- 0.3 mrad


IMU/GPS Covariance Analysis
Single Axis Tilt Error, 1sigma (mrad
GPS-CEP (m)
10
1
0.1
0.01
0.1
1
10
LN200
DQI
IMU700
IMU400
Kinematic
Differential
P-Code


Std. Dev. of Horizontal Velocity Error vs. Time for INS IMU400
Horizontal Velocity Error [RSS of σ2 of VN & VGE] (meters/sec)
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
during INS calibration (with differential GPS @ 1Hz)
0
5
10
15
20
25
Time (sec)


Std. Dev. of Horizontal Velocity Error vs. Time for INS IMU400
Horizontal Velocity Error [RSS of σ2 of VN & VGE] (meters/sec)
2
1.5
1
0.5
0
after INS calibration (with differential GPS @ 1Hz)
0
5
10
15
20
25
Time (sec)