Comments on "Federated Square Root Filter for Decentralized Parallel Processes"

In N. A. Carlson's recent article [1, p. 518, col. 1, para 1], he refers to my prior work [2] (and by direct extension its refinement in [3]) and its conceptual decentralized filtering structure ([3, Fig. 8]) as having *no theoretical justification* for the decentralized filtering aspect since *no mathematical basis* for it is offered in [2] (or [3]). It is true that the underlying theory for decentralized filters was not specifically worked out in detail *again* in [2] or [3] (however, the essence was presented in abbreviated form in [3, sect. IVC] and [2, sect. 4.3] with supporting implementation details specified in [3, sect. IV] and [2, sect. 4.2] because my primary thrust in [2 and 3] was to elucidate the recently developed failure detection amelioration aspect, to convey the new results for real-time managing of this aspect, and to show how it fit within the context of *existing* decentralized filtering as a natural melding with my prior failure detection experience (as can be gleaned from [17–20] and from the further references cited in [4–6], and from my primary military application experience in this failure detection area for submarine navigation, as specifically cited in the references and footnotes of [20]). However, the underlying theory for my approach to decentralized filtering *was* worked out in the predecessor references that I cited in [2, 3], being [7–9] here (also see [11]) and in particular [10], which Dr. Carlson and I jointly coauthored (along with Dr. Jerome Sacks), a document which originally provided all the details. As an outside consultant to Intermetrics in late 1983, Dr. Carlson also monitored a solicited Army proposal response P-7294 that I did at Intermetrics for AVRADCOM on this topic of decentralized filtering. Almost all the salient points conveyed in [1] regarding navigation applications of decentralized filters and their subsequent robustness to subsystem failures were originally made by me in that proposal as the natural culmination of my earlier investigation into these aspects in [9, Sect. 1.5 and 5], where the first conclusions were drawn on the joint utility of decentralized filters in general multisensor navigation applications. I felt no compulsion to rehash the existing theory of decentralized filtering in [2 and 3] since it had already been admirably developed and clearly reported (as it evolved and was refined) by J. L. Speyer (1979), T. S. Chang (1980), A. S. Willsky et al. (1982), and Levy et al. (1983) in a form that *is* applicable to the time-varying case encountered in navigation applications,[1] all being references cited in [1] (and also in [2, 3], where I gave proper credit and additionally cited a precedent within the abbreviated description of the principles of operation of decentralized filters in [3, sect. IV.C] and [2, sect. 4.3]). An overview explanation of how the inherent cross correlation can be taken into account and compensated in appropriately combining several local estimates to obtain the optimal global estimate ([21, pp. 185–189]) along with providing illustrative simplified low-order simulation examples for variations of this Speyer approach for navigation applications (viz., JTIDS RelNav) were offered in 1981–1982 by G. Gobbini and W. S. Widnall (and later by J. F. Kelley), respectively, in [3, ref. [136, 137, and 140]]. Widnall enjoys international renown as a seasoned navigation practitioner (e.g., [22–24]). While Carlson claims ([1, sect. 1, at end of para. 3]) that *no decentralized filter formulations have been implemented in real-time for navigation*, as apparant justification for his starting from scratch and building up the theory of decentralized filtering from first principles again. I cite four more precedents on [3, p. 101] and [3, ref. [98, 152, 197]] and in C-4 Trident SINS/ESGN submarine

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[1]Carlson asserts in [1, p. 517, para. 3] that this approach (that I find fully competitive to Carlson's and perhaps even better) was "not suitable or practical for real-time estimation of time-varying systems, due to restrictive system assumptions or large data transfer requirements" but Carlson doesn't get into any specifics on these issues in [1] (that would perhaps allow such assertions to be refuted item by item). While I have a history of being reasonably selective, discriminating, and critical both in the *failure detection arena* (e.g., [3, Table 1] and [2, Table 2-1]) and in the area of *decentralized filter formulations* (e.g., [9, Table 3-1] and [8, Table 1]) especially as they relate to navigation applications, as well as for critically reviewing reduced-order filter methodologies [25, pp. 75–83]; I have *not* encountered any restrictive assumptions in the Speyer/Chang/Willsky/Levy, et al. approach that were not ultimately loosened sufficiently in the later installments of the theoretical development. While Willsky et al. (1982) and Levy et al. (1983) chose to expedite the reporting of their new ground-breaking results for decentralized filtering by first rigorously deriving these results in their shortest time-invariant form (while explicitly indicating applicability to time-varying situations as well) and Levy et al. justified their new installment of results using the most expedient path of a *Scattering Theory* derivation; *it is well-known that Kalman filter results all carry over to apply to time-varying systems in general*, with alternative approaches existing for supplying detailed proofs (as well-known to be available and that can be filled in using any one of the seven different approaches: 1) Orthogonal Projections (in a *Hilbert Space*), 2) Recursive Least Squares, 3) Maximum Likelihood, 4) Minimum Variance, 5) Conditional Expectations, as all demonstrated in five alternative centralized filter derivations [26, ch. 7], or the two additional approaches: 6) Three Martingales (of Balakrishnan (1971)), or my personal favorite (as used for deriving decentralized filtering structures in [7, Appendix A, Sect. 2.4], and in [9, sect. 2.2, 2.3]), 7) use of the Matrix Maximum Principle). More will be said about appropriately tailoring Speyer's approach to the multisensor navigation application in the next paragraph.

navigation, where *real-time decentralized navigation filters have been implemented.*

While Speyer's original development (for command, control, communication, and identification C^3I applications) avoided the military single-point-vulnerability issue of having only a central processing node by Speyer's cross communicating so much information between each of the n participating decentralized filters in the network that *each* filtering node could fully reconstruct the global optimal estimate, I recognized in [2 and 3] that this full flexibility is *not* needed for the application of current interest involving multisensor navigation fusion in a single aircraft, so I proceeded to select for use in [2 and 3] just the minimum subset of cross communication required to support total synergistic use of *all* the available sensor measurements for a globally optimal estimate reconstruction to occur at just a *single* node, designated to be the *unification collating filter* output in [2 and 3], while each individual constituent filter in my design of [2 and 3] still correctly cover their previously assigned individual jurisdictions by providing the locally optimal estimate under their operational constraints of only being allowed to use the locally available sensor measurements. In the event of a recognized processor failure (where prescribed voting/tallying algorithms are offered in [2 and 3] within the *voter/monitoring screen* for recognizing underlying failures in real-time), these local filters still correctly perform their originally assigned function of providing locally optimal estimates at the locally designated rate and so provide a degree of robustness in their backup mode of operating singly. The results of Willsky et al. ([1, ref. [3]]) and Levy et al. ([1, ref. [4]]), respectively, provide the flexibility invoked in [2 and 3] of the n filter nodes having distinctly different subset system models and different measurement source sensors and noises (and associated analytic characterizations or representations) and even rigorously accommodate use of reduced-order models ([1, ref. [4, sect. V]]) within their particular decentralized filtering framework that I have tapped into for navigation applications. The idea of using a single collating filter within a single platform was deduced by me from Levy et al. ([1, ref. [4, Fig. 8]]) and the introduction of an intermediate *voter/monitoring screen* was my novel contribution in [2 and 3] (viz. [3, Fig. 8]; cf. Carlson's almost identical [1, Fig. 1]), which I justified there while providing details for a practical mechanization. So my decentralized filter formulation of [2 and 3] *does have an analytic mathematical basis*, as just recounted above.

Except for [1, Fig. 1] and [3, Fig. 8] having almost identical high level block diagram representations, Carlson's futuristic so-designated *type B systems* do differ fundamentally from what was offered or suggested by me for use in [2] or [3]. On a positive note, Carlson develops the square root filter and information filter form of decentralized filtering in [1], as recommended in [3, p. 105, last sentence in col. 1] to be the next logical step that is needed in decentralized filter development. A prior 1987 precedent [27] illustrates the mechanics of formulating decentralized parallel filters in square root and information form, just as Carlson has done. In a more critical vein, however, I have great apprehension concerning Carlson's *type B systems*, especially regarding the sharing of initial conditions and system process noise across n participating filters according to his weighted-linear-combination rule using the weightings [1, eq. (26)]: γ_i, where

$$\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \cdots + \frac{1}{\gamma_n} = 1,$$

and $0 \leq 1/\gamma_i \leq 1$. The main problem with use of this scheme is that no individual filter gives the correct answer (the correct answer being either the global or locally optimal estimate or conditional expectation given the measurements, as normally associated with the output of a single centralized filter). In Carlson's type B framework, the correct answer is only obtained if *all* participating decentralized filters are available and *all* participating sensor subsystems are unfailed. Thus, this is a larger computational burden to implement than use of a single centralized filter yet offers little robustness of performance in the face of processor or sensor availability failures that would delete the expected contribution of a constituent filter. Hence, Carlson's *type B systems* offer only drawbacks without any apparent ameliorating benefit as an offset. There appears to be no way (obvious or otherwise) to extend Carlson's type B approach (derived for exclusively linear systems) to the nonlinear case. The decentralized filtering formulations that I have investigated in the past [9] and which I advocate for use in [2 and 3] do not suffer from such weaknesses. The target tracking applications that I have been involved in for the last three years are inherently nonlinear and involve Kalman filter extensions and approximations embodied as extended Kalman filters (EKFs) [12]. A recent independent investigation [16] reports the details that enable use of decentralized estimators for nonlinear systems. Regarding the utility of Carlson's prior square root filter formulation [13] and its relationship to Bierman's $U - D - U^T$ formulation, correct but unflattering independent assessments can be found in [14, p. 338, col. 2, prior to sect. 2, p. 339, col. 2, next to last para., p. 342, col. 1, 2nd bullet and last para., pp. 334-5, Tables 1, 2, 3], [15, pp. 403–404, example 7.12, Tables 7.1, 7.2].

THOMAS H. KERR
Lincoln Laboratory of MIT
L-192
244 Wood St.
Lexington, MA 02173-9108

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Author's Reply

T. H. Kerr's comments address several points regarding my recent paper [1], his own prior work, and other prior work on decentralized filtering. This reply responds to his main points in approximately the order presented.

1) My recent paper referred to Kerr's work in [2] (see also [3]) as a "conceptual decentralized filter", and stated that "no mathematical basis for that filter was presented." This statement was true but incomplete, in that it did not mention Kerr's referral to prior work as the basis for his approach. If this omission has confused or misled anyone, I apologize.

2) The federated filter method developed in [1] is fundamentally different from Kerr's prior work in Intermetrics' 1983 proposal for the Army. The federated filter is based on a new information-sharing methodology for distributing and recombining information in a partitioned optimal estimator, whereas Kerr's proposal work addressed multirate filtering, singular perturbation approaches to reduced-order filtering, and stability issues in GPS/JTIDS hybrid navigation systems. The navigation system applications and fault-tolerant aspects of the federated filter described in [1] were developed independently by me, in response to Air Force objectives for the associated research effort. These features are novel, not in their concept, but in their *implementation* via the new federated filter method.

3) Kerr describes a number of previous approaches to decentralized filtering, provides prior examples where decentralized navigation filters have been implemented, and questions the reason for my deriving "the theory of decentralized filtering ... again." First, there is no one theory of decentralized filtering. Different approaches lead to different algorithms, different computation requirements, and different data transfer requirements. Second, the utility of the theory developed in the primary references cited by Kerr (Speyer [4], Chang [5], Willsky et al. [6], Levy et al. [7]), seems to lie in the eye of the beholder. Kerr refers to the theory in these papers as "admirably developed and clearly reported." I don't disagree with that statement. However, in reviewing those papers myself, my assessment was that they contain valuable ideas and useful theoretical approaches for certain problems, but that the associated data transfer requirements and/or computational burdens make them unattractive for real-time avionics system applications. Similar conclusions were reached in 1987 by an independent survey of existing decentralized filtering methods [9, 10], conducted as part of the Air Force's Common Kalman Filter (CKF) program. The pros and cons of several of those methods are detailed in [9, 10]. Perhaps Kerr has subsequently found ways to reduce the data transfer requirements and computational burdens to make those methods more practical.

My statement regarding prior decentralized methods ("none appear to have been implemented in real-time system applications, e.g., aircraft navigation systems"), is an over-generalization. I was referring specifically to *optimal* decentralized filter methods implemented as cascaded filters, and was drawing my conclusion from those methods described in the open literature (e.g., [4–8]). I did not see any of these methods being used in current integrated filter designs involving GPS (built-in local filter cascaded with second, master filter), for example.

So you didn't look further than these 4 references

I cited 5 precedents

Kerr also cites two specific decentralized filter implementations. The Trident SINS/ESGN submarine navigation filter I have not seen. (Is it an optimal partitioned estimator? Being designed for a submarine, is it practical in a high-rate aircraft environment?) The Widnall-Gobbini filter [11] employs a measurement-sharing (rather than estimate-sharing) approach aimed at JTIDS community navigation; it is quite different from the partitioned optimal estimators, or cascaded filters, of primary concern here. Incidentally, Widnall participated as a consultant, as did I, on the CKF program cited above [9, 10]. The prime contractor eventually selected the federated filter as the baseline formulation for the fault-tolerant CKF. Bierman's decentralized square root information filter [12] was also considered favorably.

different reference from what I cited. He changes the question (again)!

In summary, I myself have not seen evidence that prior, optimal cascaded filter methods have been implemented in aircraft navigation systems; however, I do not deny the possibility. With due respect to Dr. Kerr, I would be interested in seeing another paper from him that details the mathematical equations needed to implement one of these prior methods, preferably in matrix-vector form suitable for coding. Presumably, these equations would show how the apparent high computation burden, data transfer loads, and inter-filter coordination requirements of [4–7] can be overcome. Also helpful would be simulation results demonstrating filter optimality, plus quantitative measures of filter computational burden and databus loadings. This specific information would be very helpful to other researchers, in determining exactly what method is recommended

This is what my current white paper provides (in the works) i.e., in preparation

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by Kerr, and whether it is in fact practical for their own applications.

4) The next-generation "Type B" systems and the current-generation "Type A" systems of [1] differ from prior methods, including Kerr's, by virtue of their underlying basis in the new information-sharing methodology. By design, no individual local filter provides the "correct" (fully optimal) solution, nor should it. The optimal or near-optimal solution is provided by the master filter. (The local filters also provide local solutions that are only slightly noisier than locally optimal.) If a local filter/sensor failure is detected, the master filter simply stops using that local filter solution. At the same time, it adjusts the information-sharing multipliers γ_i, e.g., from n to $n-1$, when one of n local filters fails. No loss of *unfailed* local filter information occurs, except for a possible brief transient.

local type B filter has wrong initial conditions and wrong process noises yielding wrong estimates

The real benefit of the federated filter, here, is its potential ability to *detect* and *isolate* a failed sensor in the first place, particularly a "soft" (slowly growing) failure. Centralized filters sometimes fail to detect a soft failure, since they continually adjust their sensor bias states to achieve consistency across the sensor set. Measurement residuals may rise temporarily, then decay back to normal levels, as the filter adapts to the slow failure. In contrast, the federated filter allows sensor-specific errors to accumulate in the corresponding local filter solutions (between fusion resets, if any); hence, an individual sensor failure can accumulate in one local filter until its effects become large enough to be detected, via comparison with other local filter solutions.

5) With regard to computational efficiency, the federated filter almost always provides a reduced computation burden *per processor* in a multiprocessor configuration, compared with a centralized filter on one processor. This case is of primary interest in decentralized systems. However, if the federated filter components were for some reason implemented on a single processor, then its peak-cycle processing burden would usually (but not always) be greater than that of a centralized filter, while its average burden would usually (but not always) be less than that of a centralized filter. The break-even points depend upon the number of common reference-system (e.g., INS) states, the number of sensor bias states, the number of measurements per local sensor/filter, and the various update frequencies.

6) Regarding nonlinear systems, the federated filter is not intended for use with highly nonlinear systems. However, it is applicable to *linearizable* (moderately nonlinear) systems, since it can be implemented as a set of extended Kalman filters. Some caution is necessary, since the local filters may be more sensitive to nonlinearities than a centralized filter, given that they individually contain less information, and hence have larger estimation errors.

7) There is no issue, here, regarding "utility" of the triangular square root [13] versus U-D [14] filter formulations. Relative to the federated filter, block-triangular square-root matrices provide a convenient and efficient *structure* for defining the partitioned covariance operations. However, U-D *mechanizations* of that structure were recommended in [1]. For the record, the comparative assessment of the two methods by the independent source (Maybeck) cited by Kerr was actually this, in summary [15, p. 392]: "$UD^{1/2}$ corresponds directly to the covariance square root of the Carlson filter ..., and the Carlson filter in fact partially motivated this filter development .... Though similar in concept and computation to the Carlson filter, this algorithm does not require any of the $(nm+s)$ computationally expensive scalar square roots as processed in the former." I agree.

NEAL A. CARLSON
Integrity Systems, Inc.
600 Main St.
Suite 4
Winchester, MA 01890

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