

Duality Between Failure Detection and Radar/Optical Maneuver Detection

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The two distinct technical specialty areas of *failure detection in stochastic dynamical systems* and the *detection and tracking of target maneuvers by radar or optics* are identified here as two separate disciplines that have achieved many results, but generally by using different techniques. These techniques can be cross-applied to the other specialty area, which is revealed to have the same underlying mathematical formulation of the event detection problem in common.

I. UNDERLYING MODEL USED IN FAULT DETECTION

A linear model with an additional impulsive component to represent a failure, as offered in [1, eq. (1)], has been used for failure detection in dynamic systems for 20 years in this context (cf., [28]) as the standard model to represent the presence of failures (cf., failure models of [3, eqs. (1), (2)], [22, eqs. (1), (2)], [24, eqs. (41), (42)]). As recently presented [1, eq. (1)], the underlying model for failure detection applications is usually a stochastic linear system (where for continuous-time formulations, it is a stochastic ordinary differential equation (ODE); while for discrete-time formulations, as usually posed in the computer mechanization of the solution algorithm, it is a recursive stochastic difference equation) of the following form:

$$x(k+1) = \Phi(k+1, k)x(k) + w(k) + \nu\delta_{k,\theta} \quad (1)$$

$$z(k) = H(k)x(k) + v(k) \quad (2)$$

where $x(k)$ is the state at time step k describing the time evolution of the underlying system, $z(k)$ is the corresponding measurement provided by some sensor(s), and $w(k)$ and $v(k)$ are independent, zero mean, Gaussian white noises, having covariances of intensity $Q(k)$ and $R(k)$, respectively, and $x(0)$ is a

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Gaussian random vector initial condition (independent of the noises) of mean x_0 and variance P_0 . Standard technical assumptions in force are that the system of (1), (2) is either *observable* and *controllable* or at least *detectable* and *stabilizable*. A detailed consideration of controllability and observability conditions being satisfied by navigation error models is provided in [30]. Many other failure detection applications have involved models of this same form.¹

It is assumed in this discussion that an adequate modeling effort has already preceded to provide assignments of the dimension and identities of the states x , the measurement components z , and correct quantifications of the transition matrix $\Phi(k+1, k)$, the observation matrix $H(k)$, and the symmetric positive definite covariances $Q(k)$, $R(k)$, and P_0 . For failure detection applications, this modeling effort usually culminates in the specification of an appropriate Kalman filter to be used in tracking system performance. The Kalman filter model may be of reduced order or reduced complexity [25, 26] than possessed by the original model of (1), (2) so that it may be accommodated despite any imposed constraints on the computational resources available in the particular application. In some applications, the actual underlying system could more properly be modeled as a stochastic nonlinear ODE, in which case failure detection would ultimately still be applied to a refined model of the form of (1), (2) which results from the standard step of linearization of the original model, as expanded about a specified operating point. Within the above (1), there is an additional additive term representing the failure, with a random time of occurrence denoted by the unknown θ , and a random direction and magnitude in state-space denoted by the vector ν . The Kronecker delta $\delta_{k,\theta}$ in (1) is zero except when $k = \theta$ (the system time k now attaining the a priori unknown time of event occurrence) indicating that the failure has occurred. Both of the above identified unknowns θ and ν must be accounted for or estimated by some means or deciphered by an appropriate "mixed" hypothesis test in order to

¹Failure detection models of three other simple forms are offered in [24, p. 607] which, respectively involve incurring step changes in the dynamics [24, eq. (41)] and/or incurring impulsive or step changes in the measurement sensors, respectively, as [24, eq. (43) and (44)]. From navigation applications, it is known that each of these three additional apparent modeling alternatives can be accommodated within or reduced to be of the form of (1), (2) by simple modeling techniques such as by acknowledging that jumps in the derivative states (when included in the system model) correspond to step changes in the state of interest (cf., similar approach in [33] for maneuver detection) and that more detailed structure and failure mechanisms of a sensor (normally associated with (2)) such as the presence of random biases or the presence of serially time-correlated "colored" measurement noise can in fact be routinely included within an augmented dynamics model of the form of (1). Please see [31, 34, section 4.5] for explicit examples.

accomplish the *successful* detection of a failure, when it occurs.

II. AN OBSERVATION OF CROSS-APPLICABILITY BETWEEN SIMILAR SPECIALTY FIELDS

Evidently, models similar to (1), (2) for failure detection also arise in radar applications of "change detection," as in the tracking of randomly maneuvering (evading uncooperative enemy) vehicles/aircraft/missiles/satellites or in distinguishing signatures of particular radar targets before and after deploying "antennas" or other appendages thus signifying that the device has been "enabled" and is currently fully operational (or has changed its mode of operation to be more threatening). Similar concerns have caught the attention of other radar applications engineers in the past (e.g., [7, 29, 32]) and will likely continue to be of interest. The potential for fruitful cross-fertilization between the two technical specialty areas of radar event detection and failure detection motivates me to bring up this topic now.

III. SURVEYS OF ALTERNATE APPROACHES TO FAULT/FAILURE, MANEUVER, DISCRETE-EVENT DETECTION

To inform the radar practitioner about failure detection results, there are several English language surveys of failure detection developments from 1976 to the present, such as offered in [17, 20, 21, 24] which summarize results to date and then proceed to predict how the field will likely continue to evolve based on past trends. As discussed in [12, 14, 15, section 2], these surveys tend to be somewhat optimistic in predicting what can actually be achieved using these prior techniques since they sometimes extrapolate to predict what will likely be available in years to come if present trends continue. However, many successes of failure detection have already occurred in situations having fewer complications and less esoteric models than portrayed in some of the earlier surveys.

Properly or rigorously handling the above described model of (1), (2) encountered in failure detection of seeking to *detect a signal of unknown form at an unknown time of occurrence* is a *hard problem*, as attested to in the limited usefulness (as acknowledged by the author of [8]) in encountering an intractable decision threshold specification for a test statistic for even the scalar results obtained in [8] using a "jump" process or point-process formulation to model the occurrence of failure, and in its more tractable slight variation in [9] to handle increases in the variance of the drift parameter (with anticipated extensions to handling the harder problem, originally addressed in [8], of seeking to detect changes in the "drift" coefficient, that has yet to be realized using the

methodology of [9]). Several other researchers are actively looking into how to best handle problems exhibiting this structure (for example, see [5, 6]).

Reference [15] offers recent suggestions (using multiple Kalman filters) on how to best proceed in making inroads into adequately solving this thorny and challenging problem in detection and decision theory for real-time failure detection for modern avionics applications. These modern systems have tighter delay time constraints (frequently on the order of fractions of a second but sometimes up to several minutes) than were encountered in the prior Marine Inertial Navigation System application (frequently on the order of fractions of a day and sometimes several days but still a comparable small fraction of the months long mission time) for which the "two ellipsoid overlap" technique of [2, 3, 16, 18, 19, 22] was originally developed (over ten years ago). There have been several other recent accomplishments of note within this challenging analytic area of performing failure detection such as the approach of [11] using the sequential probability ratio test (SPRT) formulation of [13] as a more pliable framework for handling the "mixed" hypothesis situation of failure and event detection than was available by the 1947 formulation of SPRT in [27] that rigorously handles only "binary" hypothesis situations.

To inform the failure detection practitioner about radar maneuver detection results, [4] and [23, p. ix, session on "Detection and Estimation of Changes in Stochastic Models"] (in English) report significant Soviet inroads made in handling these types of problems. Considerations in optical tracking are provided in [29]. A comparison of the performance of various maneuver detection algorithms is offered in [35].

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