Extending Decentralized Kalman Filtering (KF) to 2-D for Real-Time Multisensor Image Fusion and/or Restoration: Optimality of Some Decentralized KF Architectures*

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Abstract

We look into both the theoretical and practical aspects of "decentralized" Kalman filtering (KF) methodology as it is combined with the methodology of existing 2-D Kalman filtering for image restoration to yield a new architecture for multisensor fusion of images in close to real-time. The outfitting of each participating imaging sensor with its own dedicated 2-D Kalman filter (raster scanned in multi-layer sync) allows a final collating filter to assemble the data from diverse imaging sensors of various resolutions into a single resulting image.

We also respond here to a recent (Jun.' 96) critical theoretical scrutiny of fundamental decentralized KF architectures which concluded that they are usually sub-optimal in general. We identify and vindicate use of a particular decentralized KF formulation as being "exact" and therefore an optimal linear estimator to be used in our approach to imaging sensor fusion (as well as being exact in other identified KF applications).

1 Introduction

As a new theoretical development [1], we pursue the idea that recent "decentralized" Kalman filter (KF) technology [2]–[9], by outfitting each participating imaging sensor with its own dedicated 2-D Kalman filter [1] can be used as the basis of a sensor fusion methodology that allows a final collating filter to assemble the data from diverse imaging sensors of various resolutions into a single resulting image that combines all the available information (in analogy to what is already routinely done in multisensor NAVIGATION [7], [30]–[49]).

The novelty is in working out the theoretical details for 2-D filtering situations (using [50]–[54] as a guide) while assuming that the image registration problem (reduction to a common scale and coordinated alignment registration) has already been independently handled [55], perhaps by hardware proximity multiplexing through a shared common aperture [perhaps using rotating mirrors] where scale of sensor scene image could have been calibrated and adjusted in a static environment beforehand) [2]. We must synchronize frame size and location of pixels of interest to be comparably located with same "raster scan" speed and size used for each to match up for different sensors. Rule for Kalman filters is that the combining of underlying measurements or sensor information can only help and never hurt.

We interpret this approach as involving several common views of the same scene, as instantaneously obtained from different sensors, with intermediate results all stacked up vertically one planar view on top of another planar view, each with its own local 2-D Kalman-like image restoration filter proceeding to raster scan (in multi-layer sync). Then applying the multi-filter combining rules from Decentralized filtering [4, Sec. 1.5], [29] to the bunch yields a single best estimate image as the resulting output and as a convenient and

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[1]Started with state of the art as it has evolved over the past 20 years for "centralized" version of 2-D KF, as surveyed in [10]–[17] in identifying the most practical implementation to yield best performance for complexity incurred in implementation and then added our novel theoretical results to it [2]–[9] to enable sensor fusion [1].

[2]The specific application which motivated this particular imaging approach and exhibits these characteristics is discussed in [1, Sec. 1, pp. 548-549] and in [49].

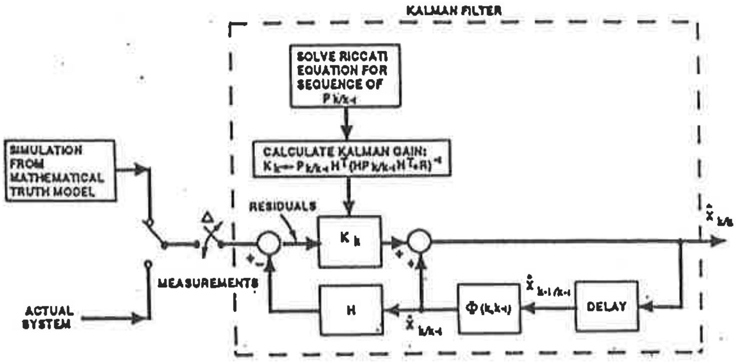


Figure 1: A Standard centralized discrete-time KF mechanization (1-D)

useful methodology to achieve sensor fusion.

While we have previously sketched an overview of the constituents of our approach to image restoration and sensor fusion and its rationale in [1], we reveal our particular 2-D mechanization equations here in Sec. 3.2 after reviewing the history of 2-D Kalman filtering for Image Restoration in Sec. 3.1 as a prologue. The Kalman filter structure of Fig. 1 is flexible, compliant, and efficient enough so that it can expedite real-time Image fusion using this new technique as has already been done in many other KF applications represented in [30]–[49], as characterized in Sec. 2.3. The purpose of this new algorithm is to compensate for motion blur and for general noise abatement while additionally reaping the benefits of alternative views of the same scene from the augmenting perspective of different sensors that tap into various complementary spans within the range of the electromagnetic spectrum.

We also respond here to a recent critical theoretical scrutiny of fundamental decentralized KF architectures [18] which concludes that most are usually sub-optimal in general. Use of a particular *exact* decentralized KF is key to the success of our sensor fusion approach, so we emphasize the benefits of this decentralized KF implementation in Sec. 2 and explain when it's *optimal* and when it's *sub-optimal* and how to correctly evaluate its actual performance in either case.

2 Decentralized KF Status

It initially seemed to be unnecessary to further revisit the existing theory of decentralized filtering in [6] and [7] (which instead focuses more on new results in failure detection and redundancy management reaped by utilizing the decentralized KF architecture) since the various important aspects of decentralized KF were already developed and clearly reported (as it evolved and was refined) by J. L. Speyer (1979), T. S. Chang (1980), A. S. Willsky et al (1982), and Levy et al (1983) in a form that is already applicable to the time-varying case needed for navigation applications. Detailed summaries had been provided earlier in [2]–[4], (and also in [6], [7], where a precedent for this particular formulation is cited as being [72] and the gist of this 1968 precedent is demonstrated within an abbreviated description of the principles of operation of decentralized filters in [7, Sec. IV.C] ([6, Sec. 4.3])).

However, since the decentalized version of Kalman filtering and its benefits are still largely unfamilar to most signal processing professionals/practicioners (viz., as his rationale for not tapping into what had already been done, [19] claims that his "new" theory of decentralized Kalman filtering needs to be worked out in its entirety from scratch since there are no prior theoretical formulations or precedents in applying it to navigation applications [dispite the existence of [5] which somehow was overlooked in [19]]), the existing theory is summarized here again for the reader's convenience while offering certain new results within the fabric of the historical context so that they can be better appreciated (as we also relate several other theoretical precedents).

An overview explanation of how the inherent cross-correlation can be taken into account and compensated in appropriately combining several local estimates to obtain the optimal global estimate (pp. 185-189 of [21]) along with providing illustrative simplified low-order simulation examples for variations of this Speyer approach for navigation applications (viz., JTIDS RelNav) were offered in 1981-82 by G. Gobbini and W. S. Widnall (and later by J. F. Kelley), respectively, in Refs. 136, 137, and 140 of [7]. Five more navigation precedents of using decentralized KF's were cited on p. 101 of [7] (as Refs. 98, 152, 197 of [7]) and in Sandia Corporation's SITAN and in C-4 Trident SINS/ESGN submarine navigation, where real-time decentralized navigation filters *have already been* implemented since at least

1976 (viz., 14 state ESGN Reset Filter and 15 state SINS Correction Filter and 7 state STAR filter for the SINS alone) in Fig. 2 from [43], being both reduced-order multi-rate cascaded filters since the existing truth model had 100+ and 34 states for the RI/Autonetics ESGN and G7-B SINS, respectively). And now there are even more decentralized KF applications [19], [20], [56], [58], [64].

While Speyer's original development (for Command, Control, Communication, and Identification C^3I applications) avoided the military single-point-vulnerability issue of having only a central processing node by Speyer's cross-communicating so much information between each of the n participating decentralized filters in the network that each filtering node could fully reconstruct the global optimal estimate (see Fig. 3); it was recognized in [6] and [7] that this full flexibility is not needed for the application of current interest involving multisensor navigation fusion in a single aircraft, so we selected just the minimum subset of cross-communication required to support total synergistic use of *all* the available sensor measurements for a globally optimal estimate reconstruction to occur at just a single node [3] (see Fig. 4), designated to be the *Unification Collating Filter* output (see Fig. 5) in [6] and [7], while each individual constituent filter in the design of Fig. 5 still correctly cover their previously assigned individual jurisdictions by providing the locally optimal estimate under any operational constraints of only being allowed to use just the locally available sensor measurements.

In the event of a recognized processor failure (where prescribed voting/tallying algorithms are offered within the *Voter/Monitoring Screen* for recognizing underlying failures in real-time), these local filters still correctly perform their originally assigned function of providing locally optimal estimates at the locally designated rate and so provide a degree of robustness in their backup mode of operating singly.

The results of Willsky et al (for the time-varying systems of Eqs. 1.5 to 1.7 in Ref. 3 of [19]) and Levy et al (Ref. 4 of [19]), respectively, provide the flexibility invoked in [6], [7] of the n filter nodes having distinctly different subset system models and different measurement source sensors and noises (and associated analytic characterizations or representations) and even rigorously accommodate use of reduced-order models (Section 5 in Ref. 4 of [19]) within their particular decentralized filtering framework that was tapped into for navigation applications. The idea of using a single collating filter within a single platform was deduced from Levy et al (see Fig. 8 in Ref. 4 of [19]) and the introduction of an intermediate *Voter/Monitoring Screen* was the major contribution in [6] and [7] (cf., Fig. 8 of [7] to Carlson's almost identical Fig. 1 of [19]), which was justified there while providing details for a practical mechanization in [7, Sec. IV.B].

[3] Another back-up would be to have a second *Unification Collating Filter* on a different processor as just one controlled level of redundancy.

Carlson's futuristic so-designated *Type B Systems* differ fundamentally from what was offered for use in [6] or [7]. On a positive note, Carlson develops the square root filter and information filter form of decentralized filtering in [19], as recommended in [7, p. 105, last sentence in column 1] to be the next logical step that is needed in decentralized filter development. [61] constitutes a prior 1987 precedent illustrating the mechanics of formulating decentralized parallel filters in square root and information filter form, just as Carlson has done. In a more critical vein, however, there may be some concerns regarding Carlson's *Type B Systems*, related especially to the sharing of initial conditions and system process noise [4] across n participating filters according to his weighted-linear-combination rule using the weightings [19, Eq. 26]: γ_i, where $\frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \cdots + \frac{1}{\gamma_n} = 1$, and $0 \le \frac{1}{\gamma_i} \le 1$.

The main problem with use of this scheme appears to be that no individual filter gives the correct answer (the correct answer being either the global or even a locally optimal estimate or conditional expectation given only the local measurements, as normally associated with the output of a single reduced-order Kalman filter). In Carlson's Type B framework, the correct answer is only obtained if all participating decentralized filters are available and all participating sensor subsystems are unfailed. Thus, this is a larger computational burden to implement than use of a single centralized filter yet offers little robustness of perfor-

[4] At least Carlson's formulation has process noise intensity Q. The four alternative decentralized formulations offered by O. E. Drummond for distributed or decentralized multi-target tracking are devoid of *any* process noise. When process noise is absent, alternative decompositions are much easier to obtain. One of the few applications that can be represented without an underlying Q is the post-boost phase of reentry vehicle tracking for the "Star Wars" Strategic Defense Initiative, which is in fact Drummond's application. Upon actual reentry back into the atmosphere, Q should again be present to account for turbulence in RV lift and drag so other KF decompositions are then needed.

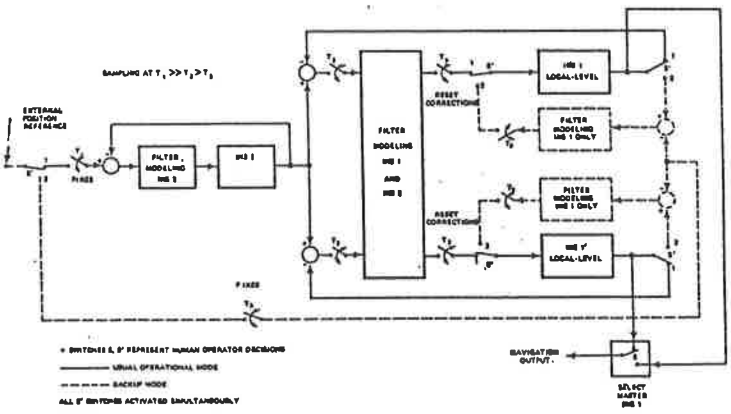


Figure 2: Overview of the INS1/INS2 System.

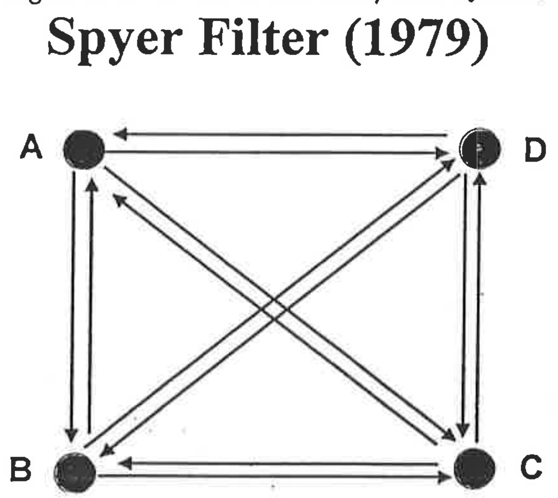


Figure 3: Globally optimal KF estimates can be reconstructed at each node.

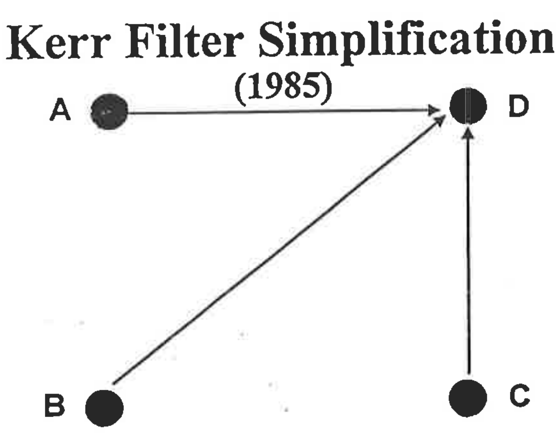


Figure 4: Globally optimal KF estimates can be reconstructed ONLY at node D.

mance in the face of processor or sensor availability failures that would delete a constituent filter's expected contribution. Hence, Carlson's *Type B Systems* appear to offer only drawbacks without any apparent ameliorating benefit as an offset in a trade-off.

2.1 Decentralized filtering benefits

Standard techniques for quantifying the computer burden associated with implementing alternative filter mechanizations have been refined over the years and typically involve assessments of algorithm operation counts, the corresponding algorithm cycle times as determined for the target machine, and allotments of program and stored memory (J. Mendel, *IEEE Trans. on AC*, Dec. '71). Precedents in applying such tallies are offered in [2]–[4] for several alternative decentralized filtering formulations. The utility of such considerations is quantitatively illustrated below in arguing the case favoring implementing a Kalman filter in distributed form on two (or more) processors rather than as a standard single large filter on one processor (that is more susceptible to being throughput limited). Some early approximations (A. N. Joglekar and J. D. Powell, *AIAA G&C*, Aug.'73) are a historical filtering approach to navigation data compression with simplifications to conserve on-line computer resources being its rationale.

As an example of the benefits to be reaped in going to a decentralized filter formulation, for two separate GPS and JTIDS filters of dimension 12 and 15, respectively, the advantage of two over one larger 19 state unified filter is obtained from the ratio of the total number of required operations as:

$$\frac{(12)^3 + (15)^3}{(19)^3} = \frac{5103}{6859} = 0.74$$

or a 26 percent reduction in the total number of operations to be performed during each filter cycle even though the INS gyro drift-rate states are modeled twice. Unfortunately, a slight 2 percent increase in required computer memory allotment for data is indicated by

$$\frac{(12)^2 + (15)^2}{(19)^2} = \frac{369}{361} = 1.02\,.$$

However, the large benefit in throughput as the major consideration in such applications appears to be well worth the slight penalty.

The case favoring two separate filters is even more pronounced or compelling when considering an alternative state selection corresponding to two filters of state size 12 and 18 versus a single 22 state filter since calculations of the above form indicate *savings* to be achieved in both the number of operations (equivalent to algorithm cycle time of processing a filter measurement) and computer data memory required as, respectively, 30 percent and 3 percent.

If two separate digital processors are used, parallel processing of each of the two filters on different machines provides the advantage that the system is only limited by the slower speed of the single larger filter (of 15 or 18 states). In comparison, the smaller filter of 12 states can proceed through 6 Kalman filter measurement processing cycles in the same time that a larger unified 22 state filter could complete only one cycle, as indicated by the following ratios:

$$\frac{(22)^3}{(12)^3} = \frac{10648}{1728} = 6.16\,.$$

The conclusion is that a unified single filter will limit processing throughput and hinder full utilization of the GPS measurements available. The above arguments are graphically portrayed in Fig. 6. Precedents for filter sizes depicted here and specifics of states utilized are provided in [7, Table III].

A fairly straight forward generalization to three concatenated but nested filters (i.e., the states of the consecutive filter models are nested by perhaps an allowable similarity transformation) operating at decreasing sampling rates of fast/medium/slow is represented as the middle diagram of Fig. 7. The approach for accomplishing this task is roughly merely a back-to-back repetition of the two filter technique already worked out in [4] but applied separately to each two-filter pair of, first, the fast/medium, then to the medium/slow filter pair (with generalization to Fig. 5 being immediate).

2.2 Commonality between four decentralized KF formulations

"Partitioned" Kalman filters have been used for RV tracking since the initial development of SAFEGUARD anti-missile system for first city then silo perimeter defense (pre-dating SDI) as an approximation to alleviate the computational burden (Brown, Cohen, et al, *EASCON*, 1977) before SAFEGUARD was abandoned, and "partitioning" has been advocated by Daum and Fitzgerald

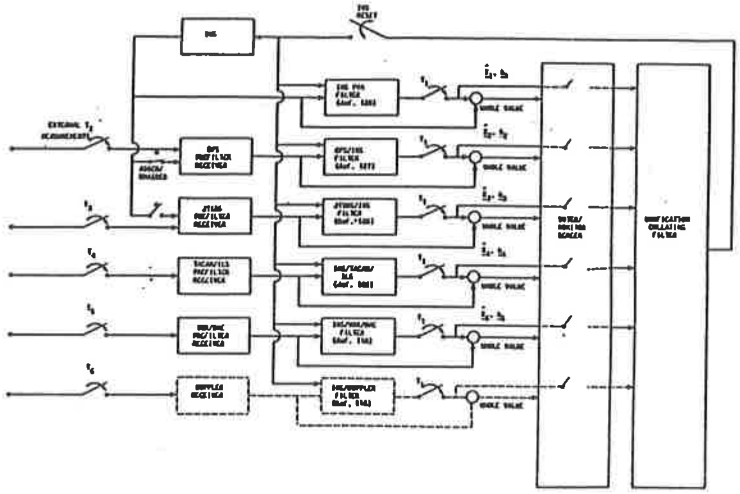


Figure 5: Decentralized Semi-autonomous Multisensor Navigation (SMN) filter to enhance failure detection/isolation and to ease reconfiguration

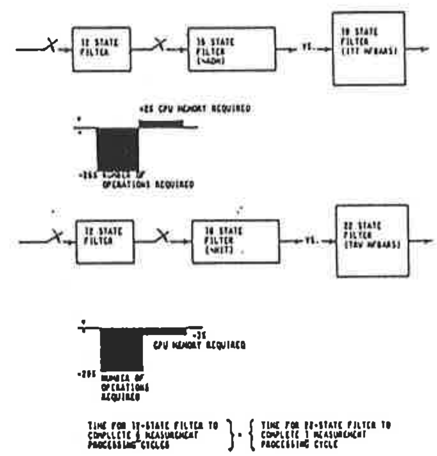
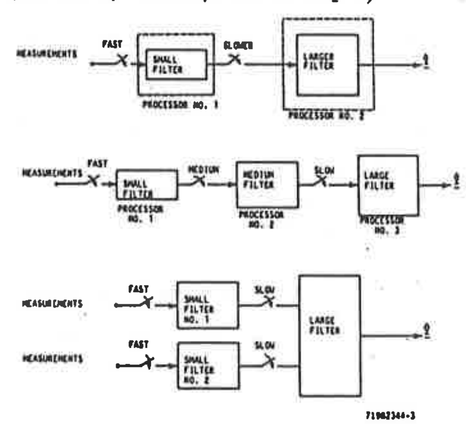


Figure 6: Benefits of two filters over one federated filter (for GPS/JTIDS/INS example)



•Properly chosen non-optimum filtering procedures can have performance that is close to optimum, while offering better throughput/failsafe redundancy.

Figure 7: New approaches to Kalman filtering via parallel processing

(*IEEE Trans. on AC*, 1983). Even though Steve Rogers had initially offered a refinement for "partitioned" filters (*IEEE Trans. on AC*, 1986), Rogers now identifies (in NAECON, 1988 and in [22]) the high likelihood for partitioned filter instability and the divergence that frequently occurs within this approximation. Hence, "partitioning" is no longer as lucrative a technique as it once was considered to be (as a type of cascaded decentralized filtering) in target tracking. Notice that these are yet another variant of decentralized KF.

Three independent teams of investigators use the same *alternate form* [5] of the "centralized" KF implementation equations that they select as a jumping off point for generalization [6] to the following diverse applications:

- Multi-sensor camera data fusion for robotics and/or telerobots [60];
- Target tracking using data from non-geographically co-located sensors with coupling via noisy communication lines [61] (cf., [59]);
- Multi-sensor *integrated* navigation [63].

All of the above three investigations now use the alternate KF form (depicted here in Table 1) for computing the Kalman gain K_k (in [60, Eq. 7], [61, Eq. 7b], [63, Eq. 45]) involving use of the covariance update $P_{k|k}$ instead of using the usual predicted covariance $P_{k|k-1}$. While these references do display their final mechanization equations, they don't show its derivation, which can be elusive and not obvious so we offer our derivation below to expose important details.

The alternate form for centralized KF mechanization (serving as the fundamental stepping stone or jumping off point in [60]-[63] for eventual generalization to the decentralized filtering case) has a theoretical twist that is utilized within this structure, as reviewed next. The measurement records collected by the multiple 1-D decentralized sensors ($i = 1, \ldots, N$) can be summarized in aggregate block form as measurements:

$$z(k) = \left[z_1^T(k), \ldots, z_N^T(k)\right]^T , \tag{1}$$

and as an effective observation matrix:

$$H(k) = \left[H_1^T(k), \ldots, H_N^T(k)\right]^T , \tag{2}$$

and as an effective additive measurement noise:

$$v(k) = \left[v_1^T(k), \ldots, v_N^T(k)\right]^T , \tag{3}$$

with the further assumption that the zero mean white Gaussian measurement noises across partitions (i.e., between sensors as a consequence between different planar views for the 2-D generalizations to come) are uncorrelated (from sensor to sensor) so that the associated covariance intensity matrices are of the form

$$E\left[v(k)v^T(k)\right] = \text{blockdiagonal}\{R_1, \ldots, R_N\} . \tag{4}$$

Similarly, let each sensor's local system model consist of the same [7] $n \times 1$ state vector in common throughout, of the form

$$x_i(k+1) = \Phi_i(k+1,k)x_i(k) + w_i(k) , \tag{5}$$

with a suitably tailored (specialized) $m_i \times 1$ vector measurement model for sensor i of the form

$$z_i(k) = H_i(k)x_i(k) + v_i(k) , \tag{6}$$

so each local filter, using the alternate KF formulation, is expressible as in Table 1.

The vehicle or contrivance for linking up these results for eventual decentralized filtering is the formation of the centralized $H^T(k)R^{-1}(k)H(k)$ as

$$H^T(k)R^{-1}(k)H(k) = \sum_{j=1}^{N} H_j^T(k)R_j^{-1}(k)H_j(k) . \tag{7}$$

Now from Table 1, the covariance update formula for the i^{th} sensor may be rewritten as

$$P_i^{-1}(k|k) - P_i^{-1}(k|k-1) = \underbrace{H_i^T(k)R_i^{-1}(k)H_i(k)}_{\text{LHS covariance info broadcast from each local sensor } i} \tag{8}$$

[5] The distinction being in how the Kalman gain is calculated (cf., Tables 1 and 2 of [1]).

[6] All identically reminiscent of the earlier structural result of [72].

[7] Here is the assumption that matches up with the identical assumption in [25] and [26] that harkens back to Bar-Shalom's revelation (*AES*, 1986) that two tracks from different sensors tracking the same target are correlated because of the common underlying process noise maneuver model. [25] obtains nice closed-form solutions using $\alpha - \beta$ filters and analytic solutions to Sylvester's Equation (i.e., a non-symmetric Lyapunov Matrix equation) for the case of just two sensors. [26] gets into heavy use of pseudo-inverses in decentralized multisensor target tracking. The theory and mechanics of pseudo-inverse construction are quite familiar [34], [35]; however, the necessity of their use in this target tracking application remains to be seen.

	PROPAGATE STEP	UPDATE STEP
COVARIANCE	$P_i(k\|k-1) = \Phi_i(k,k-1)P_i(k-1\|k-1)\Phi_i^T(k,k-1) + Q_i(k)$	$P_i(k\|k) = [P_i^{-1}(k\|k-1) + H_i^T(k)R_i^{-1}(k)H_i(k)]^{-1}$
FILTER GAIN		$K_i(k) = P_i(k\|k)H_i^T(k)R_i^{-1}$
FILTER	$\hat{x}_i(k\|k-1) = \Phi_i(k,k-1)\hat{x}_i(k-1\|k-1)$	$\hat{x}_i(k\|k) = \hat{x}_i(k\|k-1) + K_i(k)(z_i(k) - H_i(k)\hat{x}_i(k\|k-1))$

Table 1: "Alternate" Kalman Filter Implementation/Mechanization Equations (as separate local filters not yet combined)

and similarly for the aggregate global centralized covariance update as

$$P^{-1}(k|k)-P^{-1}(k|k-1) = \sum_{j=1}^{N} H_j^T(k)R_j^{-1}(k)H_j(k), \tag{9}$$

which may now be reexpressed (by substituting Eq. 8 in Eq. 9) as

$$P^{-1}(k|k) - P^{-1}(k|k-1) = \sum_{j=1}^{N} \left[P_j^{-1}(k|k) - P_j^{-1}(k|k-1)\right] , \tag{10}$$

as an equation for the global covariance update in terms of the summation of local entities (consisting of $n(n+1)/2 + n = n(n+3)/2$ floating point variables) originally calculated at the j^{th} sensor (j=1 to N) and broadcast via a communication network to a processor node that is tasked with collating all the local information into a global *best answer*.

Another benefit of block decomposition of the aggregate centralized form is in exposing the following equivalence that exists:

$$H^T(k)R^{-1}(k)z(k) = \sum_{j=1}^{N} H_j^T(k)R_j^{-1}(k)z_j(k) . \tag{11}$$

Another simplifying contrivance is the observation from the covariance update, known as Joseph's form, which is known to be mathematically equivalent to

$$P(k|k) = [I - K(k)H(k)]\, P(k|k-1) \tag{12}$$

(cf., [60, Eq. 20], [61, Eq. 9], [63, Eq. B11]). Then post-multiplying throughout Eq. 12 above by $P^{-1}(k|k-1)$ yields

$$[I - K(k)H(k)] = P(k|k)P^{-1}(k|k-1) . \tag{13}$$

Now by taking the estimate update equation as

$$\hat{x}(k|k) = \hat{x}(k|k-1)+K(k)(z(k)-H(k)\hat{x}(k|k-1)) , \tag{14}$$

we further have that

$$\hat{x}(k|k) = \hat{x}(k|k-1) + K(k)(z(k) - H(k)\hat{x}(k|k-1)) =$$
$$[I - K(k)H(k)]\,\hat{x}(k|k-1) + P(k|k)H^T(k)R^{-1}(k)z(k) \tag{15}$$

and substituting for $[I - K(k)H(k)]$ from Eq. 13 and pre-multiplying throughout by $P^{-1}(k|k)$ yields

$$\begin{aligned}
&P^{-1}(k|k)\hat{x}(k|k) \\
&= P^{-1}(k|k)P(k|k)P^{-1}(k|k-1)\hat{x}(k|k-1) \\
&\quad +P^{-1}(k|k)K(k)z(k) \\
&= P^{-1}(k|k-1)\hat{x}(k|k-1) \\
&\quad +P^{-1}(k|k)P(k|k)H^T(k)R^{-1}(k)z(k) \\
&= P^{-1}(k|k-1)\hat{x}(k|k-1) + H^T(k)R^{-1}(k)z(k) \\
&= P^{-1}(k|k-1)\hat{x}(k|k-1) + \textstyle\sum_{j=1}^{N} H_j^T(k)R_j^{-1}(k)z_j(k) ,
\end{aligned} \tag{16}$$

and, by now pre-multiplying Eq. 16 throughout by $P(k|k)$, yields the fundamental estimation update expression:

$$\hat{x}(k|k) = P(k|k) \left[P^{-1}(k|k-1)\hat{x}(k|k-1) + \textstyle\sum_{j=1}^{N} H_j^T(k)R_j^{-1}(k)z_j(k)\right] . \tag{17}$$

as an equation for the global state update in terms of the summation of local entities originally calculated at the i^{th} sensor (i=1 to N) and broadcast via a communication network to a processor node that is tasked with collating all the local information into a global best answer.

By a derivation route and arguments identical to that presented for Eqs. 11 to 17, we obtain a local state estimation equation of a form similar to that of Eq. 17 for each local sensor as

$$\hat{x}_i(k|k) = P_i(k|k) \left[P_i^{-1}(k|k-1)\hat{x}_i(k|k-1) + H_i^T(k)R_i^{-1}(k)z_i(k)\right] , \tag{18}$$

or, rearranged to be

$$P_i^{-1}(k|k)\hat{x}_i(k|k) - P_i^{-1}(k|k-1)\hat{x}_i(k|k-1) = \underbrace{H_i^T(k)R_i^{-1}(k)z_i(k)}_{\text{LHS estimate info broadcast from each local sensor } i} \tag{19}$$

In conclusion, the final architecture for centralized globally optimal estimates obtainable from the indicated info broadcast on the network from each local sensor i is derivable from Eq. 19 substituted into Eq. 17 as

$$\hat{x}(k|k) = P(k|k) \left[P^{-1}(k|k-1)\hat{x}(k|k-1) + \sum_{j=1}^{N}\{P_j^{-1}(k|k)\hat{x}_j(k|k) - P_j^{-1}(k|k-1)\hat{x}_j(k|k-1)\}\right] \tag{20}$$

(cf., [60, Eq. 26], [61, Eq. 16b], [63, Eq. 51]) to be used along with the covariance update of Eq. 10, rearranged as

$$P(k|k) = \left[P^{-1}(k|k-1) + \left[\sum_{j=1}^{N}\{P_j^{-1}(k|k) - P_j^{-1}(k|k-1)\}\right]^{-1}\right]^{-1} = \left[A^{-1} + B^{-1}\right]^{-1} = A\left[A + B\right]^{-1} B \,, \tag{21}$$

(cf., [60, Eq. between Eqs. 17 and 18], [61, Eq. 17b], [63, Eq. 52]) [8] which is recognized to be of the form of a triple $n \times n$ matrix inversion, where operations counts for each of these inversions is merely n^3. Reiterating, reference [60] recommends that each local sensor node i broadcast two pieces of critical information at each designated synchronous time step k being (1) the $n \times n$ matrix difference $\{P_i^{-1}(k|k) - P_i^{-1}(k|k-1)\}$ and (2) the $n \times 1$ vector difference $\{P_i^{-1}(k|k)\hat{x}_i(k|k) - P_i^{-1}(k|k-1)\hat{x}_i(k|k-1)\}$, which have now already been demonstrated above to be equivalent to transmitting the normally expected natural info on $H_i(k)$, $R_i^{-1}(k)$, and $z_i(k)$ (respectively, of dimension $m_i \times n$, $m_i \times m_i$, and $m_i \times 1$). However, since the above matrix difference arising in Eq. 21 is symmetric, one only needs to actually transmit $n(n+1)/2$ entries of the matrix rather than n^2 at each time step k as a considerable savings.

The obvious perceived benefit of the above formulation is *structural consistency* (independence of particular local m_i) in the collating update architecture of Eqs. 20 and 21. Although, unstated in [60]-[63], an even greater perceived benefit of the architecture being offered is that for sensors that fail to report by the designated collation time for time-step k (due to possible failures, battle damage, overly delayed message packets, pruned outlier readings, etc.) the summation can still take place (using info from those sensors that do report) to yield *the best there is* with the collection of local information available at the time!

This author had previously cautioned (or reminded) the estimation community in [65, p. 944, Eq. 47] not to make the mistake of using the simpler version of the discrete-time Kalman Covariance Update Equation:

$$P_{k|k} \;=\; [I - K_k H_k] P_{k|k-1} \,,$$

when the following (so-designated Joseph's form) should be used instead [9]:

$$P_{k|k} \;=\; [I - K_k H_k] P_{k|k-1} [I - K_k H_k]^T + K_k R K_k^T \,.$$

Da was also pursuing the reduced-order filtering problem in [29] but, unfortunately, so many of the existing *so-called reduced-order filtering methodologies* currently being employed are flawed, with enumerations detailed in [35, pp. 79-82] and independently confirmed [71]. The author Da had some interesting new ideas on use of optimal and simpler sub-optimal "combining rules" (for combining the local estimation results from separate local filters to obtain the globally optimal estimate as an outcome) that are of interest in what is discussed later. The present author has also obtained more expedient sub-optimal combining rules in [4, Sec. 1.5].

2.3 More candidates for decentralized KF reformulation

APPLICATIONS: Over the past thirty years, Kalman filters (KF) have been used in telephone line echo-cancelers, missiles, aircraft, ships, submarines, tanks that shoot-on-the-run, air traffic control (ATC) radars, defense and targeting radar, Global Position System (GPS) sets, and other standard navigation equipment. Historically, Kalman filters have even been used in modeling to account for the deleterious effect of human reaction times on the ultimate performance

[8] Notice that there are slight discrepancies between what is summarized here and what was offered at comparable steps in [63] so, strictly speaking, the approach of [63] is not *identical* to that of [60] and [61] even if *most* of the particulars are the same. Similarly, [61] looks further into an information filter formulation and a square root filter formulation after it has passed through these same primary results that are revealed here to be in common with the other two approaches. However, these further formulations are rather routine KF variations.

[9] The former doesn't yield the correct covariance associated with using a reduced-order suboptimal Kalman gain K_k while the latter does!

of a control system having a man-in-the-loop. In recent years, GPS/Kalman filter combinations in conjunction with laser disk-based digital map technology is being considered for use in future automobiles (as well as in ships using displays rather than paper charts) to tell the driver/pilot where he is and how to get where he wants to be. Commercial products as well as military vehicles and platforms rely on Kalman filters. Computers are used to implement Kalman filters and to test out their performance (assess the speed of response and the accuracy of their estimates) beforehand in extensive simulations. Indeed, Kalman filter variations are now being used for supervised learning in some Neural Networks. (For more examples, see special March 1983 issue of *IEEE Transactions on Automatic Control*, Vol. AC-28, No. 3 entirely devoted to other less well known applications of Kalman filters. Also see March 1982 NATO AGARDograph No. 256 and February 1970 NATO AGARDograph No. 139 for standard applications.) Those that may benefit from a decentralized reformulation are depicted in Fig. 8.

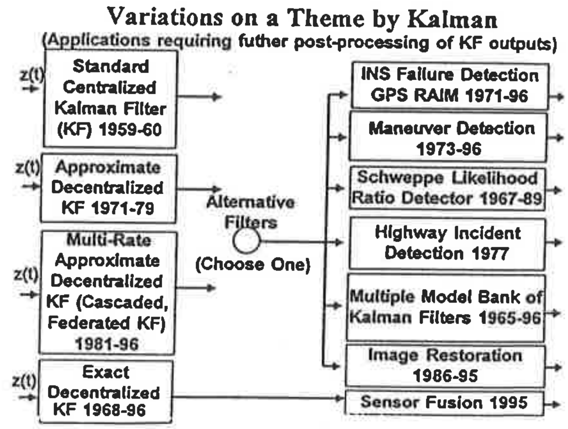


Figure 8: Variations on a theme by Kalman

Besides the exciting areas of multi-sensor/multi-target tracking in clutter (related to optimal resource allocation solved by invoking appropriate generalizations of *Munkres*, or the *Hungarian*, or the *Jonker-Volgenant-Castenon* (JVC) algorithm from *Operations Research*), we perceive the cutting edges of Kalman filter theory and technology to be along the following five application fronts enumerated below:

1. (Now that hardware has caught up to make such endeavors practicable [10]) application of parallel "Bank-of-Kalman-Filters" (see Fig. 9 where each has a different underlying system model matched or representing a different hypothesized underlying situation) with global probability assessments of each filter possibly coinciding exactly with the true situation [currently prevailing and from which the only measurements are availed throughout] being automatically calculated on-line as an integral part of this totally rigorous methodology for linear systems. [As conceived of in 1965 but only relatively recently pursued for actual

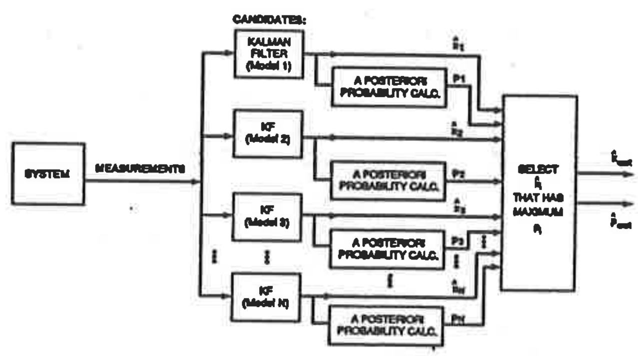


Figure 9: Multiple Model of Magill (MMM): N alternative filters, each with its distinctly different system models, vying to match the true (unknown) system as it progresses through its likely operating regimes (characterized a priori by analysts), with associated on-line computation of probabilities of each being correct so that a tally is available to decide which one (choice of a "winner" varying with time) offers the best match

[10] The ideal natural computational framework for both this Bank-of-Kalman-filters algorithm and our sensor fusion architecture would be synchronized multi-processing on a machine with several processor boards (one per filter and one for each probability calculator/maximization selector) proceeding in parallel using a pre-emptive multi-processing operating system, such as Microsoft's Windows NT or IBM's OS/2.

use within the last 7 years in IR, GPS, Radar, and multi-target Sonar applications with significant extensions being provided in the last six years by Y. Bar-Shalom, H. Blom, and X.-R. Li.]

2. Extension beyond standard 1-D random process Kalman filter scenario to a 2-D Markov Random Field (MRF) for image processing situations for motion blur reduction and noise abatement [11].

3. "Decentralized Kalman filter formulations" tailored to applications where there is a natural fit and an obvious advantage to be exploited, such as in:

   - On-line "Failure Detection, Identification, and Reconfiguration" (FDIR) in analytic (virtual) and actual redundancy management of complementary and supplementary sensors within coherently focused NAV Systems [7];
   - "Sensor Fusion" of frame-synchronized scenes for multiple sensor-dedicated Kalman Filters using optimal and sub-optimal combining schemes [1];
   - Decentralized detection [combining decentralized Kalman filtering and hierarchical decision processes] [26];
   - Generalization of Kalman filter-based Schweppe likelihood ratio [47] to multiple filter situation, and to nonlinear system situation (involving non-Gaussian noise statistics).

4. A fully rigorous extension of the standard Kalman filter beyond those systems routinely described by ODE's to systems described by partial differential equations (PDE's) [using infinite dimensional Banach-space derivation techniques rather than the Hilbert-space techniques that Rudy Kalman used in originally discovering/revealing/uncovering the Kalman Filter structure for optimal estimation] has materialized and matured in the last 25 years [12];

5. Another application scenario for decentralized Kalman filtering is [33] in monitoring the state of a distributed application [as a prelude to control compensation] for a flexible large space structure (LSS), such as arises within the proposed Space Station Freedom and within other orbiting platforms, where these techniques will again be useful in the face of likely time-shared tasking of onboard computational resources and a distributed implementation architecture, making it necessary for multiple sensor measurement data to vie for access to communication links so that it may be conveyed in a timely fashion to relatively scare computational resources concentrated at various places across the structure (where decisions have to be made on which subset of the totality of measurements actually gets through to be processed).

2.4 Recent Criticisms of Cascaded, Federated KF

The discussion (ION, June '96) by Larry J. Levy (APL/JHU) reported in [18] is extremely well written and insightful and is *almost* a "paragon of perfection" that <u>should</u> be read by everyone seriously interested in this field. Levy correctly identifies in the Introduction that the Federated filter is a generalization of Cascaded filters and is optimal (i.e., equivalent to the centralized filter) when the full global state is modeled in each local filter and the Master filter is run at the (fast) data rates of the local filters (but untenable in practical online implementations). Levy also correctly identifies how to perform dual state covariance analysis (for the reduced-order filters present) to obtain a proper evaluation of the actual covariance of estimation error (for linear systems).

However, Levy taps into the methodology of Minimum Variance Reduced-Order (MVRO) for actual numerical evaluation in his examples (instead of perhaps using one of the more trustworthy reduced-order covariance analysis methodologies [69]). Regarding Levy's use of the MVRO methodology, several authors have already independently discussed the fact that it is unreliable as an evaluation methodology (and also say exactly why in [70], [35], [34], and especially [71] (Draper Laboratory)). Now regarding the novelty of the dual state evaluation idea, Prof. Ren Da (currently of Ohio State University, Columbus, OH, Center for Mapping, previously at GNC, in CA) introduced this three state augmenting approach

[11] Image fusion applications exist in machine vision and in medicine (ultrasound, X-Ray, NMR/NMI) as well as in military Surveillance/Reconnaissance (Lidar, millimeter wave radar imagery, IR, UV, TV).

[12] These are appropriate in monitoring lakes, rivers, and streams for impurities and contaminating effluents [thermal and chemical], where the conduction type PDE is the more accurate mathematical description of the true situation.

for covariance analysis of reduced-order decentralized filters in his Ph.D thesis ~ 1990 and in [29].

TeK Associates further observes here that the Federated filter is also optimal (i.e., equivalent to the centralized filter) when the full global state is modeled in each local filter and all the local filters are run at the synchronized common (slow) data rate of the Master filter. However, the resulting INS errors will consequently be larger in this case of a slow rate (as an Upper Bound # 1 on estimation error), while the scenario described in the prior sentence is a Lower Bound # 1 (but is an unrealistically heavy computational burden that precludes on-line use). Notice that all other asynchronous measurement processing rates typifying normal Cascaded filter and Federated filter use yield results that fall somewhere between these two upper and lower bounds (when all filters utilize the full truth model). There is no constraint imposed on Kalman filters that measurements must be periodic (however implementation is easier when measurements are received periodically) and the SINS/ESGN submarine application utilizes fixes of opportunity whenever they are conveniently accessible (so, for other applications, it shows us how to accommodate aperiodic fixes from a number of different navaid sensors [31], [32]).

A further observation here is that when using the actual prescribed reduced-order filter models througout the Cascaded or Federated filter decomposition or partitioning in conjunction with an adequate dual filter reduced-order filter covariance evaluation methodoloy [69] of augmenting the truth model states with the Master filter states, along with the states of every participating local filter, then the resulting INS errors will consequently be larger in this case of all processing taking place at the slow Master filter rate (as an Upper Bound # 2 on estimation error), while the scenario just described yields Lower Bound # 2 when all processing takes place at the rate of the fastest local filter (but is again an unrealistically heavy computational burden that precludes on-line use).

Notice that all other asynchronous measurement processing rates typifying normal Cascaded filter and Federated filter use with their respective specified reduced-order filter states yield results that fall between these two Upper and Lower Bounds # 2 (without requiring all filters to use the same filter model in common being the distinction between this more realistic situation and that of the earlier paragraph for UB and LB # 1). So we can now use covariance analysis to properly assess the accuracy hit that we tollerate in order to get timely managable througput in using realistic decentralized filters for linear systems! Nonlinear systems are handled by linearizing to also utilize this theory but prudence usually requires realistic representative Monte-Carlo simulations to demonstrate that system behavior is reasonable and as expected (as necessary but not sufficient proof).

Moreover, another TeK Associates' observation here is to remind the NAV community that properly handling NAV resets (i.e., deterministic corrective control actions such as torquing local-level gyros) in multi-filter scenarios can also be deduced from the submarine scenario: use low quality resets available from local filters at the high rate (but keep track of them by propagating each through its appropriate system transition matrix and integrated to access the effect at the current time), then when a higher quality reset is available from the Master filter, remove the effect of the earlier intermediary resets donated by the local filters and just use that provided by the Master filter. Repeat the process, starting with frequent resets from the fast local filters again until replaced by a Master reset.

3 Status of 2-D KF

3.1 History of 2-D KF for image restoration

Generalizations of standard 1-D random process evolving in time or indexed on a single time variable (isomorphic to the real line so that it is *totally ordered* for simply distinguishing past from present from future [i.e., for any t_1 and t_2, either $t_1 < t_2$, or $t_1 = t_2$, or $t_1 > t_2$] and having a standard unique definition of causality) have already been extended to 2-D [73] for Input/Output realizations. Early approaches to 2-D modeling usually invoked non-symmetric half-plane (NSHP) type causality merely for simplicity and convenience [50], [53].

The following representative milestones are recounted in briefly summarizing the generalization of Kalman filter formulations from 1-D to 2-D:

- Although Eugene Wong [10] alerts the reader in the mid 1970's and raises their level of consciousness to appreciate the difficulty of this problem (since the 2-D planar index of a random field can't be *totally ordered* for a clear unambiguous delineation of what's past, present, and future as can be done for the real line [as occurs for the time index of a random process]; however, the 2-D plane can be *partially ordered* but partial orderings are not unique and are also not wholly satisfying since there are several viable candidates that are reasonable to use but all have ambiguous "past", "present" (being a set rather than being a mere point, as occurs with a random process), and "future" defined, depending on which partial ordering convention is invoked). While [10] originally doesn't extend much hope for immediate resolution, a few years later he reports substantial progress in this area [11], [12].

- In the 1980's, Howard Kaufman along with his students and colleagues blazed an impressive development trail in further generalizations of 2-D Kalman filters specifically for image restoration applications [13]–[17]. In particular:

  - Quoting [13]: "it is established that for typical autoregressive signal models with nonsymmetric half-plane support, the dimension of the state size to be used within the Kalman filter is approximately equal to the product of the image model order and the pixel width of the image."
  - Quoting [16]: " a parallel identification and restoration procedure is described for images with symmetric noncausal blurs. It is shown that the identification problem can be recast as a parallel set of one dimensional ARMA identification problems. By expressing the ARMA models as equivalent infinite-order AR models (sic) **[the present author takes issue with this limiting claim and clarifies why in the first bullet in [1, Sec. 2.3] (based on [38], [39]) as a minor improvement]**, an entirely linear estimation procedure can be followed."
  - Quoting [17]: "it is established that an EKF for on-line parameter Identification was found to be unsuitable for blur parameter identification (sic) **[the present author takes issue with this limiting claim and clarifies why in the second bullet in [1, Sec. 2.3] (based on [68]) as a minor improvement]** because of the presence of significant process noise terms that caused large deviations between the predicted pixel estimates and the true pixel intensities."
  - Quoting [15]: "model-based segmentation and restoration of images is performed. It was assumed that space-variant blur can be adequately represented by a collection of L distinct point-spread functions, where L is a predefined integer. (*The 'Multiple Model of Magill' (MMM)*) bank of parallel Kalman filters was applied to this problem." **See Sec. 2.3 for more about MMM.**
  - Quoting [17]: "it is revealed that image restoration based upon unrealistic homogeneous image and blur models can result in highly inaccurate estimates with excessive ringing. Thus it is important at each pixel location to restore the image using the particular image and blur parameters characteristic of the immediate local neighborhood."

3.2 Our version of 2-D KF

The equation for 2-D optimal linear estimation of a scalar partial differential equation (PDE) system $\Psi(x,y)$ described over the x,y plane with boundary over the interval $[0, y_f]$ by:

$$\frac{\partial \Psi(x,y)}{\partial x} = \frac{\partial^2 \Psi(x,y)}{\partial y^2} + w(x,y) \,, \tag{22}$$

with boundary condition:

$$\alpha \Psi(x,y) = \frac{\partial \Psi(x,y)}{\partial y} \text{ at } y=0 \text{ and at } y = y_f \tag{23}$$

for scalar $\alpha > 0$ and with $w(x,y)$ being additive Gaussian white process noise in the plane of positive semi-definite intensity $Q(x,y)$ and scalar sensor measurements:

$$z(x,y) = H\,\Psi(x,y) + v(x,y) \,, \tag{24}$$

with $v(x,y)$ being additive Gaussian white measurement noise in the plane of positive definite intensity $R(x,y)$, then the associated Riccati PDE to be solved as part of optimal linear KF estimation of linear systems described by Eq. 22 is

$$\frac{\partial P(x,y,t)}{\partial x} = \frac{\partial^2 P(x,y,t)}{\partial y^2} + \frac{\partial^2 P(x,y,t)}{\partial t^2} - \int_y P(x,t,x')HR^{-1}HP(x,y,x')\,dx' + Q(y,t) \,, \tag{25}$$

with corresponding boundary conditions:

$$\alpha P(x,y,t) = \frac{\partial P(x,y,t)}{\partial y} \text{ at } y=0 \text{ and at } y=y_f\ , \tag{26}$$

$$\alpha P(x,t,y) = \frac{\partial P(x,t,y)}{\partial t} \text{ at } y=0 \text{ and at } y=y_f\ , \tag{27}$$

and

$$P(x_o,t,y) = S(t,y) \text{ an initial condition.} \tag{28}$$

These can be solved by using rectangular discretization over the 2-D plane or by using the by now well-known Finite Element technique of Fix and Strang for specifying meshes and utilizing *PDEase* code or **MatLab's** [13] PDE code (both available on a PC). We generally follow the results pioneered by Kaufman for a single sensor but for real-time use we advocate synchronizing parallel processing using one processor for each local sensor filter and a final one for the Unification Collating Filter. Compare our equations to perspectives offered in [28, Ch. 7] and [27]. Our Image Combining Rule is the 2-D analog of Eqs. 20 and 21. Simulations to date have only been with a single processor to merely demonstrate proof of concept and images to date have been *Lena* at different resolutions with different levels of additive Gaussian White Noise superimposed to corrupt the image. We welcome a real data test case!

[13] One technical problem that we did encounter was with **MatLab**'s new capability to isolate level-crossing instant of either constant or specified time-varying thresholds with almost infinite precision. This **MatLab** capability actually exists only for completely deterministic situations since the underlying algorithms are predictor/corrector-based which are stymied when noise [albeit pseudo-random noise (PRN)] is introduced in the simulation. The presence of noise has been the bane of all but the coarsest and simplest of integration methodologies since the earliest days of digital simulation. However, engineering applications where threshold comparisons are crucial usually include the presence of noise, as in detection (i.e., is the desired signal present or just noise) in radar or communications, in Kalman filter-based failure detection or maneuver detection [46], or in peak picking as it arises in sonar processing [36] and in image processing [74]. Other problems with calculation of matrix functions using matrix Signum functions, as occurs in some MatLab routines, are elucidated in (1) Barnett, S., "Comments on 'The Matrix Sign Function and Computation in Systems'," *Applied Mathematics and Computation*, Vol. 4, pp. 277-279, 1978; (2) Barrand, A. Y., "Comments on 'The Numerical Solution of $A^TQ + QA = -C$'," *IEEE Trans. on Automatic Control*, Vol. AC-24, No. 4, pp. 671-672, Aug. 1977. Also (3) Petkov, P. H., Christov, N. D., Konstantinov, M. M., "On the Numerical Properties of the Shur Approach for Solving the Matrix Riccati Equation, *System Control Letters*, Vol. 9, No. 3, pp. 197-201, 1987 for weakness in using the Shur method.

References

[1] Kerr, T. H., "Extending Decentralized Kalman Filtering (KF) to 2-D for Real-Time Multisensor Image Fusion and/or Restoration," *Signal Processing, Sensor Fusion, and Target Recognition V*, Proceedings of SPIE Conference, Vol. 2755, Orlando, FL, pp. 548-564, 8-10 April 1996.

[2] Kerr, T. H., "Stability Conditions for the RelNav Community as a Decentralized Estimator-Final Report," Intermetrics, Inc. Report No. IR-480, Cambridge, MA, 10 August 1980.

[3] Kerr, T. H., and Chin, L., "A Stable Decentralized Filtering Implementation for JTIDS RelNav," *Proceedings of IEEE Position, Location, and Navigation Symposium (PLANS)*, Atlantic City, NJ, pp. 318-329, 8-11 December 1980.

[4] Kerr, T.H., and Chin, L., "The Theory and Techniques of Discrete-Time Decentralized Filters," in *Advances in the Techniques and Technology in the Application of Nonlinear Filters and Kalman Filters*, edited by C.T. Leondes, NATO Advisory Group for Aerospace Research and Development, AGARDograph No. 256, Noordhoff International Publishing, Lieden, pp. 3-1 to 3-39, 1981.

[5] Carlson, N. A., Kerr, T. H., Sacks, J. E., "Integrated Navigation Concept Study," Intermetrics Report No. IR-MA-321, 15 June 1984.

[6] Kerr, T. H., "Decentralized Filtering and Redundancy Management Failure Detection for Multi-Sensor Integrated Navigation Systems," *Proceedings of the National Technical Meeting of the Institute of Navigation (ION)*, San Diego, CA, 15-17 January 1985.

[7] Kerr, T. H., "Decentralized Filtering and Redundancy Management for Multisensor Navigation," *IEEE Trans. on AES*, Vol. AES-23, No. 1, pp. 83-119, Jan. 1987 (minor corrections appear on p. 412 of May and on p. 599 of July 1987 issues of same journal) **Received 1987 IEEE AES Barry Carlton Award.**

[8] Kerr, T. H., "Comments on 'Estimation Using a Multirate Filter'," *IEEE Transactions on Automatic Control*, Vol. AC-34, No. 3, p. 384, March 1989.

[9] Kerr, T. H., "Comments on 'Federated Square Root Filter for Decentralized Parallel Processes'," *IEEE Trans. on AES*, Vol. AES-27, No. 6, pp. 946-949, Nov. 1991.

[10] Wong, E., "Recursive Filtering for Two Dimensional Random Fields," *IEEE Trans. on Information Theory*, pp. 84-86, January 1975.

[11] Wong, E., Tsui, E. T., "One-Sided Recursive Filters for Two-Dimensional Random Fields," *IEEE Trans. on Information Theory*, pp. 596-599, Sept. 1977.

[12] Wong, E., "Recursive Causal Linear Filtering for Two-Dimensional Random Fields," *IEEE Trans. on Information Theory*, pp. 50-59, January 1978.

[13] Angwin, D., Kaufman, H., "Image Restoration Using a Reduced Order Kalman filter," *Proceedings of ICASSP '88*, NY, pp. 1000-1003, 11-14 April 1988.

[14] Kaufman, H., Tekalp, A. M., "Survey of Estimation Techniques in Image Restoration," *IEEE Control Systems Magazine*, Vol. 11, No. 1, pp. 16-24, January 1991.

[15] Tekalp, A. M., Kaufman, H., and Woods, J., "Edge-Adaptive Kalman Filtering for Image Restoration with Ringing Suppression," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. ASSP-37, No. 6, pp. 892-899, June 1989.

[16] Biemond, J., Kaufman, H., "Identification and Restoration Using Parallel Kalman Filters," *Proceedings of 26^{th} IEEE Conf. on Decision and Control*, Los Angeles, CA, pp. 1198-1203, 9-11 December 1987.

[17] Koch, S., Kaufman, H., Biemond, J., "Restoration of Spacially Varying Blurred Images Using Multiple Model-Based Extended Kalman Filters," *IEEE Trans. on Image Processing*, Vol. 4, No. 4, pp. 520-523, April 1995.

[18] Levy, L. J., "Suboptimality of Cascaded and Federated Kalman Filters," *Proceedings of the Institute of Navigation 52nd Annual Meeting: Navigational Technology for the 3rd Millennium*, Cambridge, MA, pp. 399-407, 19-21 June 1996.

[19] Carlson, N. A., "Federated Square Root Filter for Decentralized Parallel Processes," *IEEE Trans. on Aerospace and Electronic Systems*, Vol. AES-26, No. 3, pp. 517-525, May 1990.

[20] Carlson, N. A., "Federated Kalman Filter Simulation Results," *Navigation: Journal of the Institute of Navigation*, Vol. 41, No. 3, pp. 297-321, Fall 1994.

[21] Gobbini, G. F., *Relative Ranging by means of passive ranging*, Ph.D. thesis, Dept. of Aeronautics and Astronautics, MIT, Aug. 1981.

[22] Rogers, S. R., "Instability of a Decoupled Tracking Filter," *IEEE Transactions on Automatic Control*, Vol. AC-34, No. 4, pp. 469-471, April 1989.

[23] DiFilippo, D. J., and Widnall, W. S., "Evaluation of a Kalman Filter for SAR Motion Compensation," *Proceedings of Position, Location, and Navigation Symposium (PLANS)*, Atlantic City, NJ, pp. 259-268, Dec. 1988.

[24] Iyengar, S. S., Presard, L., Min, H. (eds.), *Advances in Distributed Sensor Integration: Application and Theory*, Prentice-Hall PTR, Upper Saddle River, NJ, 1995.

[25] Saha, R. K., "Effect of Common Process Noise on Two-Sensor Track Fusion," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 4, pp. 829-835, July-August 1996.

[26] Pao, L. Y., "Measurement Reconstruction Approach for Distributed Multisensor Fusion," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 4, pp. 842-847, July-August 1996.

[27] Šebek, M., "Polynomial Solution of 2-D Kalman Bucy Filtering Problem," *IEEE Transactions on Automatic Control*, Vol. AC-37, No. 10, pp. 1530-1532, October 1992.

[28] Sage, A., *Optimum Systems Control*, Prentice-Hall, Englewood Cliffs, NJ, 1968.

[29] Da, R., "The Error Covariance Analysis Algorithms for Suboptimal Decentralized Kalman Filters," *AIAA Journal of Guidance, Control, and Dynamics*, pp. 909-913, Sept.-Oct. 1993.

[30] Kerr, T. H., "Preliminary Quantitative Evaluation of Accuracy/Observables Trade-off in Selecting Loran/NAVSAT Fix Strategies," TASC Technical Information Memorandum TIM-889-3-1, Reading, MA, December 1977 (Confidential).

[31] Kerr, T. H., "Improving C-3 SSBN Navaid Utilization," TASC Technical Information Memorandum TIM-1390-3-1, Reading, MA, August 1979 (Secret).

[32] Kerr, T. H., "Modeling and Evaluating an Empirical INS Difference Monitoring Procedure Used to Sequence SSBN Navaid Fixes," *Proceedings of the Annual Meeting of the Institute of Navigation*, U.S. Naval Academy, Annapolis, Md., pp. 10-19, 9-11 June 1981 (Selected for reprinting in *Navigation: Journal of the Institute of Navigation*, Vol. 28, No. 4, pp. 263-285, Winter 1981-82).

[33] Kerr, T. H., "Further Comments on 'Optimal Sensor Selection Strategy for Discrete-Time Estimators'," *IEEE Trans. on AES*, Vol. 31, No. 3, pp. 1159-1166, June 1995.

[34] Kerr, T. H., "The Proper Computation of the Matrix Pseudo-Inverse and Its Impact in MVRO Filtering," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-21, No. 5, September 1985.

[35] Kerr, T. H., "Computational techniques for the matrix pseudoinverse in minimum variance reduced-order filtering and control," in *Control and Dynamic Systems-Advances in Theory and Applications,* Vol. XXVIII: Advances in Algorithms and computational Techniques for Dynamic Control Systems, Part 1 of 3, C. T. Leondes (Ed.), Academic Press, N.Y., pp. 57-107, 1988.

[36] Kerr, T. H., "Functional and Mathematical Structural Analysis of the Passive Tracking Algorithm (PTA)," Intermetrics Report No. IR-MA-208, Cambridge, MA, 25 May 1983, for NADC.

[37] Kerr, T. H., "Streamlining Measurement Iteration for EKF Target Tracking," *IEEE Trans. on AES,* Vol. AES-27, No. 2, pp. 408-420, Mar. 1991 (minor correction in Nov. 1991 issue).

[38] Kerr, T. H., "Emulating Random Process Target Statistics (using MSF)," *IEEE Transactions on Aerospace and Electronic Systems,* Vol. AES-30, No. 2, pp. 556-577, April 1994.

[39] Kerr, T. H., *Multidimensional Shaping Filter Formulations for Vector Random Process Modeling Using Matrix Spectral Factorization,* Project Report PA-500, MIT Lincoln Laboratory, Lexington, MA, 27 Mar. 1989.

[40] Kerr, T. H., "Assessing and Improving the Status of Existing Angle-Only Tracking (AOT) Results," *Proceedings of the International Conference on Signal Processing Applications & Technology (ICSPAT),* Boston, MA, pp. 1574-1587, 24-26 Oct. 1995.

[41] Kerr, T. H., "A Critique of Neural Networks as Currently Exist for Control and Estimation," *Proceedings of the International Conference on Signal Processing Applications & Technology (ICSPAT),* Boston, MA, pp. 1434-1443, 24-26 Oct. 1995.

[42] Kerr, T. H., "Numerical Approximations and Other Structural Issues in Practical Implementations of Kalman Filtering," a chapter in *Approximate Kalman Filtering,* edited by Guanrong Chen, World Scientific, London, pp. 193-222, 1993.

[43] Kerr, T. H., "Failure Detection Aids for Human Operator Decisions in a Precision Inertial Navigation System Complex," *Proceedings of Symposium on Applications of Decision Theory to Problems of Diagnosis and Repair,* Keith Womer (editor), Wright-Patterson AFB, OH: AFIT TR 76-15, AFIT/EN, Oct. 1976, sponsored by the local Dayton Chapter of the American Statistical Association, Fairborn, Ohio, June 1976.

[44] Kerr, T. H., "Statistical Analysis of a Two Ellipsoid Overlap Test for Real-Time Failure Detection," *IEEE Trans. on Autom. Contr.,* Vol. AC-25, No. 4, pp. 762-773, Aug. 1980.

[45] Kerr, T. H., "False Alarm and Correct Detection Probabilities Over a Time Interval for Restricted Classes of Failure Detection Algorithms," *IEEE Transactions on Information Theory,* Vol. IT-28, No. 4, pp. 619-631, July 1982.

[46] Kerr, T. H., "On Duality Between Failure Detection and Radar/Optical Maneuver Detection," *IEEE Trans. on AES,* Vol. AES-25, No. 4, pp. 581-583, July 1989.

[47] Kerr, T. H., "An Analytic Example of a Schweppe Likelihood Ratio Detector," *IEEE Trans. on Aerospace and Electronic Systems,* Vol. AES-25, No. 4, pp. 545-558, July 1989.

[48] Kerr, T. H., "Status of CR-Like Lower bounds for Nonlinear Filtering," *IEEE Trans. on AES,* Vol. AES-25, No. 5, pp. 590-601, Sept. 1989 (Author's reply in Vol. AES-26, No. 5, pp. 896-898, Sept. 1990).

[49] Kerr, T. H., "Use of GPS/INS in the Design of Airborne Multisensor Data Collection Missions (for Tuning NN-based ATR algorithms)," the Institute of Navigation *Proceedings of GPS-94,* Salt Lake City, UT, pp. 1173-1188, 20-23 Sept. 1994.

[50] Andrews, H. C., and Hunt, B. R., *Digital Image Restoration,* Prentice-Hall, Englewood Cliffs, NJ, 1977.

[51] Dudgeon, D. E., and Mersereau, R. M., *Multidimensional Digital Signal Processing,* Prentice-Hall, Englewood Cliffs, NJ, 1984.

[52] Bose, N. K., *Multidimensional Systems Theory,* D. Reidel Publishing Co., Boston, 1985.

[53] Pratt, W. K., *Digital Image Processing,* 2^{nd} Edition, John Wiley & Sons, Inc., NY, 1991 (originally 1978).

[54] Baxes, G. A., *Digital Image Processing: Priciples and Applications,* John Wiley & Sons, Inc., NY, 1994.

[55] Salt, E., and Yuam, Y.-X., "On the Accuracy of Approximations in the Error Analysis of Registration Systems," *IEEE Trans. on AES,* Vol. AES-27, No. 6, pp. 922-924, Nov. 1991.

[56] Covino, J. M., and Griffiths, B. E., "A New Estimation Architecture for Multisensor Data Fusion," *PROCEEDINGS of SPIE (Society of Photo-Optical Instrumentation Engineers),* J. Wade and A. Tuchman (Eds.), Volume 1478, pp. 114-125, Orlando, FL, 2-3 April 1991.

[57] Roy, S., and Iltis, R. A., "Decentralized Linear Estimation in Correlated Measurement Noise," *IEEE Trans. on AES*, Vol. AES-27, No. 6, pp. 939-941, Nov. 1991.

[58] Wei, M., and Schwarz, K. P., "Testing a Decentralized Filter for GPS/INS Integration," *Proceedings of IEEE Position Location and Navigation Symposium (PLANS)*, pp. 429-435, December 1990.

[59] Longo, M., Lookabaugh, T. D., and Gray, R. M., "Quantization for Decentralized Hypothesis Testing Under Communication Constraints," *IEEE Transactions on Information Theory*, Vol. IT-36, No. 2, pp. 241-255, March 1990.

[60] Durrant-Whyte, H. F., Rao, B. Y. S., and Hu, H., "Toward a Fully Decentralized Architecture for Multi-Sensor Data Fusion," *Proceedings of 1990 IEEE International Conference on Robotics and Automation*, pp. 1331-1336, Cincinnati, OH, 13-18 May 1990.

[61] Roy, S., Hashemi, R. H., and Laub, A. J., "Square Root Parallel Kalman Filtering Using Reduced-Order Local Filters," *IEEE Trans. on AES*, Vol. AES-27, No. 2, pp. 276-289, Mar. 1991.

[62] Hong, L., "Centralized and Distributed Multisensor Integration with Uncertainties in Communication Networks," *IEEE Trans. on AES*, Vol. AES-27, No. 2, pp. 370-379, Mar. 1991.

[63] Hong, L., "Adaptive Distributed Filtering in Multicoordinated Systems," *IEEE Trans. on AES*, Vol. AES-27, No. 4, pp. 715-724, July 1991.

[64] Covino, J. M., and Griffiths, B. E., "A New Estimation Architecture for Hierarchical Command and Control Systems," *slide presentation at local Boston IEEE Control Systems Society*, Lincoln Laboratory, 12 February 1992.

[65] Kerr, T. H., "A Constructive Use of Idempotent Matrices to Validate Linear Systems Analysis Software," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-26, No. 6, pp. 935-952, Nov. 1990 (minor corrections in same journal, Vol. AES-27, No. 6, pp. 951-952, Nov. 1991).

[66] Ioannou, P. A. , and Datta, A., "Robust Adaptive Control: A Unified Approach," *Proceedings of the IEEE*, Vol. 79, No. 12, pp. 1736-1768, December 1991.

[67] Bar-Shalom, Y., Li, X.-R., and Chang, K. C., "Non-Stationary Noise Identification with the Interacting Multiple Model Algorithm," *Proceedings of 5^{th} International Symposium on Intelligent Control*, A. Meystel, J. Herath, S. Gray (eds.), Los Alamitos, CA, pp. 585-589, 5-7 September 1990.

[68] Ljung, L., "Asymptotic Behavior of the Extended Kalman Filter as a Parameter Estimator for Linear Systems," *IEEE Trans. on Autom. Contr.*, Vol. AC-24, No. 1, pp. 36-50, Feb. 1979.

[69] Duiven, E. M., "Suboptimal Linear Filtering," *AIAA Journal of Spacecraft and Rockets*, Vol. 11, No. 3, pp. 196-198, 1974.

[70] Asher, R. B., Herring, K. D., Ryles, J. C., "Bias, Variance, and Estimation Error in Reduced Order Filters," *Automatica*, Vol. 12, pp. 514-517, Nov. 1976.

[71] Setterlung, R., "New insights into minimum variance reduced order filters," *AIAA Journal of Guidance, Control, and Dynamics*, 1988.

[72] Klinger, A., "Information and Bias in Sequential Estimation," *IEEE Trans. on Autom. Contr.*, Vol. AC-13, No. 1, pp. 102-103, Feb. 1968.

[73] Bisiacco, M., Fornasini, E., and Zampieri, S., "Realization and Partial Realization of 2-D Input/Output Maps," *New Trends in Systems Theory*, Proceedings of Joint Conference at Universita di Genova-Ohio State University, G. Conte, A. M. Perdon, B. Wyman (eds.), pp. 120-127, Birkhauser, 9-11 July 1990.

[74] Zakhor, A., Oppenheim, A. V., "Reconstruction of Two-Dimensional Signals from Level Crossings," *Proceedings of the IEEE*, Vol. 78, No. 1, pp. 31-55, Jan. 1990.

[75] Azimi-Sadjadi, M. R., "New Results in Strip Kalman Filtering," IEEE Trans. on Circuits and Systems, Vol. 36, No. 6, pp. 893-897, June 1989.

[76] Azimi-Sadjadi, M. R., "Estimation and Identification for 2-D Block Kalman Filtering," IEEE Trans. on Signal Processing, Vol. 39, No. 8, pp. 1885-1889, Aug. 1991.