John Anthes, RF Monolithics, Dallas, Texas

OOK, ASK and FSK Modulation in the Presence of an Interfering signal

This paper discusses three popular modulation schemes in the presence of an interfering signal. For the purpose of our discussion OOK modulation (On/Off Key) is the special case of ASK (Amplitude Shift Key) modulation where no carrier is present during the transmission of a zero. FSK modulation (Frequency Shift Key) is commonly believed to perform better in the presence of interfering signals. However, it is usually more difficult and expensive to implement. This paper will review the three modulation types and develop a mathematical model for the prediction of error due to interference. This is then followed by laboratory test results.

OOK modulation is a very popular modulation used in control applications. This is in part due to its simplicity and low implementation costs. OOK modulation has the advantage of allowing the transmitter to idle during the transmission of a "zero", therefore conserving power. The disadvantage of OOK modulation arises in the presence of an undesired signal. As the proliferation of control and data communication apparatus increases, so does the aggravation of not being able to communicate. The three modulation types can be graphically represented on a two dimensional ortho-normal plot, sometimes referred to as a signal diagram. Consider a set of two basis vectors φ_1 and φ_2. The signal diagrams for OOK, ASK and FSK are shown in figures 1, 2 and 3 respectively. Each diagram represents the additive noise with a dashed line circle around the signal.
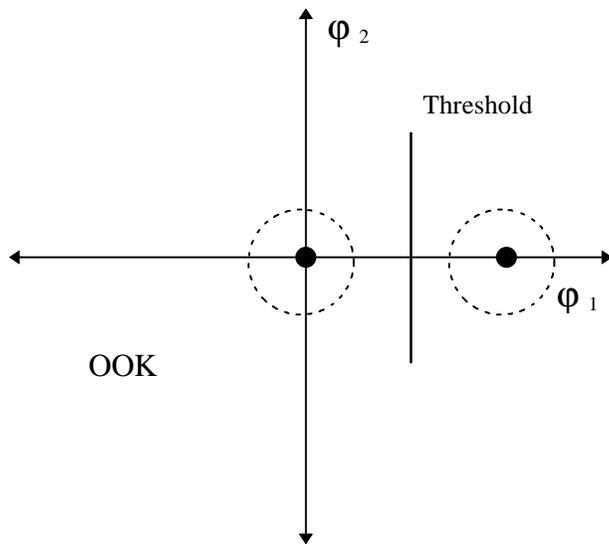
φ_2

Threshold

φ_1

OOK

Figure 1
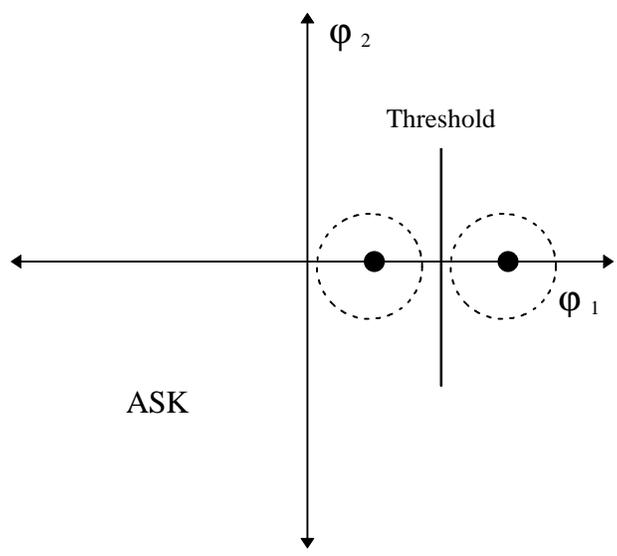
φ_2

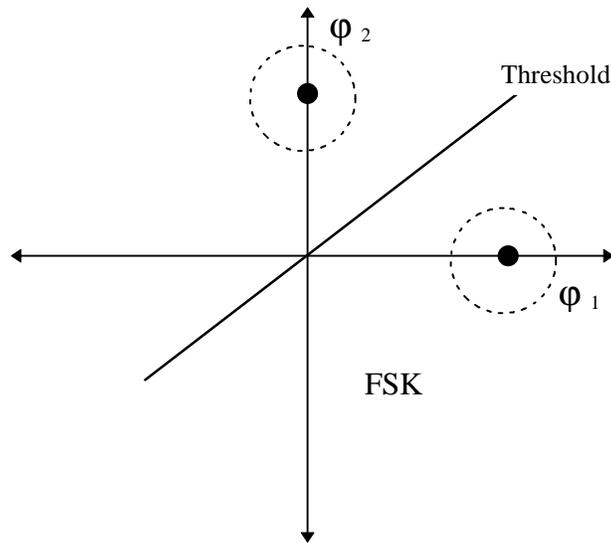Threshold

φ_1

ASK

Figure 2

Figure 3

From these diagrams it is apparent that the choice of the threshold setting is critical to the performance of the receiver. Both OOK and ASK receivers require an adaptable threshold or an automatic gain control (AGC) in order to ensure an optimal threshold setting. The FSK modulation does not usually require this because it incorporates a limiter that keeps the signal envelope amplitude constant over the useful dynamic range. The ASK and OOK receivers used for the tests in this paper employed a log amp detector with an averaging bit slicer as shown in the figure below:
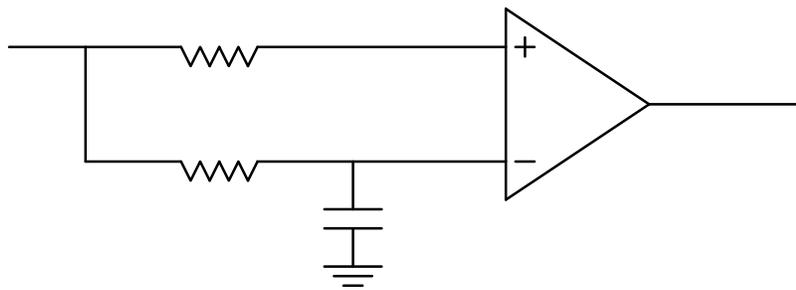

Figure 4

This circuit will ensure that the threshold is set between the signal levels of a "0" and a "1" transmission. The above circuit works well as long as the data received is reasonably D.C. balanced.

PROBABILITY OF ERROR FOR OOK AND FSK

There is an abundance of information available in literature discussing the probability of error for each of these modulation types[1]. In the case of OOK modulation, the probability density of the envelope, when no signal is present, is the Rayleigh distribution. When a signal is present, the density is Rician. The total probability of error is determined by two possible error conditions. The probability of a "1" being sent and the receiver mistaking it for a "0" (a miss) and the probability of a "0" being sent and the receiver detecting a "1" (false alarm). The total probability of error is defined in the formula below:

$$
P_\varepsilon = \frac{1}{2} \cdot \int_0^{V_t} p1(r)\, dr + \frac{1}{2} \cdot \int_{V_t}^{\infty} p0(r)\, dr \tag{1}
$$

The variable V_t is the threshold where the two probability density functions intersect. The functions p1 and p0 are the Rician probability density function and the Rayleigh density function respectively. The variable r is the random variable of the envelope detector's output. It is assumed that the likelihood that a "1" or a "0" is transmitted is 1/2. The evaluation of the above formula yields the approximate probability of error for OOK modulation using envelope detection:

$$
P\varepsilon ask_{SNR} := \frac{1}{2} \cdot \left(1 + \sqrt{\frac{1}{\pi \cdot SNR}}\right) \cdot \exp\left(-\frac{SNR}{4}\right) \tag{2}
$$

The SNR variable in the above formula is defined as the signal to noise ratio. Likewise, the probability of error can be calculated for a receiver system using noncoherent FSK modulation. The system is modeled with two matched filters centered at f_1 and f_2 with envelope detectors summed to a decision circuit. The following expression yields the probability of error for noncoherent FSK modulation.

$$
P\varepsilon fsk_{SNR} := \frac{1}{2} \cdot \exp\left(-\frac{SNR}{4}\right) \tag{3}
$$

The plot below shows both functions plotted together. The conclusion is that there is very little difference in the probability of error between OOK modulation and noncoherent FSK modulation.

$Pe\,ask_{SNR}$
———

$Pe\,fsk_{SNR}$
- - - - -

dB_{SNR}
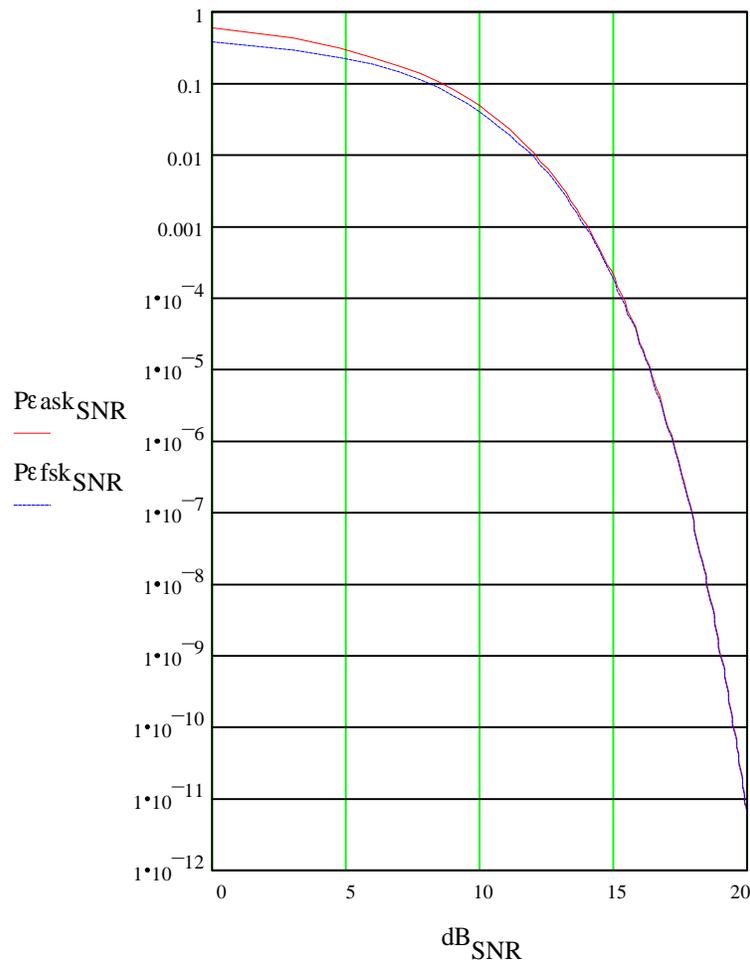
Plot 1

PROBABILITY OF ERROR WITH AN INTERFERING SIGNAL (OOK)

For the purpose of simplicity, the Gaussian distribution will be used to develop the models that follow. The Gaussian distribution is an accurate model for coherent detection methods. Coherent detection offers approximately 1 dB improvement over the noncoherent detection methods at a reasonable signal to noise ratio. The figure below illustrates the OOK modulation in the presence of an interfering signal.
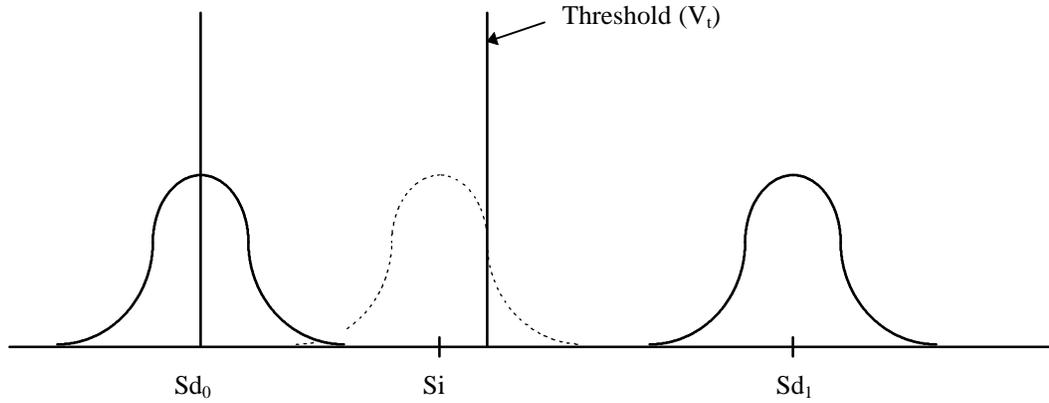
Figure 5

Sd_0 and Sd_1 are probability distributions of the desired signal for when a "0" and a "1" are being transmitted, respectively. Si is the probability distribution of an interfering signal. We will assume that the probability of a "1" being transmitted, for both the desired signal and the interfering signal, is .5 and that the desired signal is independent of the interfering signal. It is also assumed that the detection method is with an envelope detector, which implies that the largest signal at any time is being compared to the threshold. The possible errors and the probability of the condition occurring are stated below:

Condition Transmitted	Probability	Receiver Error
Sd = "1"; Si = "0" or "1"	.5	Receiver chooses a "0"(miss)
Sd = "0"; Si = "0"	.25	Receiver chooses a "1"(false alarm)
Sd = "0"; Si = "1"	.25	Receiver chooses a "1"(jammed)

(Table 1)

The model that will be developed will reference the interfering signal level to the desired signal level. The Gaussian probability density function for a "1" being transmitted with a signal level of Sd_1 and a variance of σ^2 is:

$$Psd_1 := \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma^2}} \cdot \exp\left[-\frac{\left(V_t - Sd_1\right)^2}{2 \cdot \sigma^2}\right] \qquad (4)$$

In order to allow the use of standard reference tables, the above function can be normalized with a zero mean and a unit variance.

$$P(z) := \frac{1}{2 \cdot \pi} \cdot \exp\left(-\frac{z^2}{2}\right) \qquad (5)$$

The integral below determines the probability that a random variable z, with an arbitrary mean Sd_1 and variance σ^2, is less than the threshold (V_t):

$$P_{miss} := \frac{1}{\sqrt{2 \cdot \pi}} \cdot \int_{-\infty}^{\frac{V_t - Sd_1}{\sigma}} \exp\left(-\frac{z^2}{2}\right) dz \qquad (6)$$

The above expression defines the probability of a "miss". The transmitter transmitted a "1" and the receiver declared it to be a "0". The variable V_t, which is the threshold, has previously been stated to be between the "1" and "0" levels, or 1/2 of Sd_1. The variables σ^2, and Sd_1^2 are equal to the average power of the noise and the power of the desired signal when a "1" is transmitted, respectively. Similar probability functions can be written for the case when the desired signal is a "0" and the receiver declares a "1" (False alarm);

$$P_{fa} := \frac{1}{\sqrt{2 \cdot \pi}} \cdot \int_{\frac{V_t}{\sigma}}^{\infty} \exp\left(-\frac{z^2}{2}\right) dz \qquad (7)$$

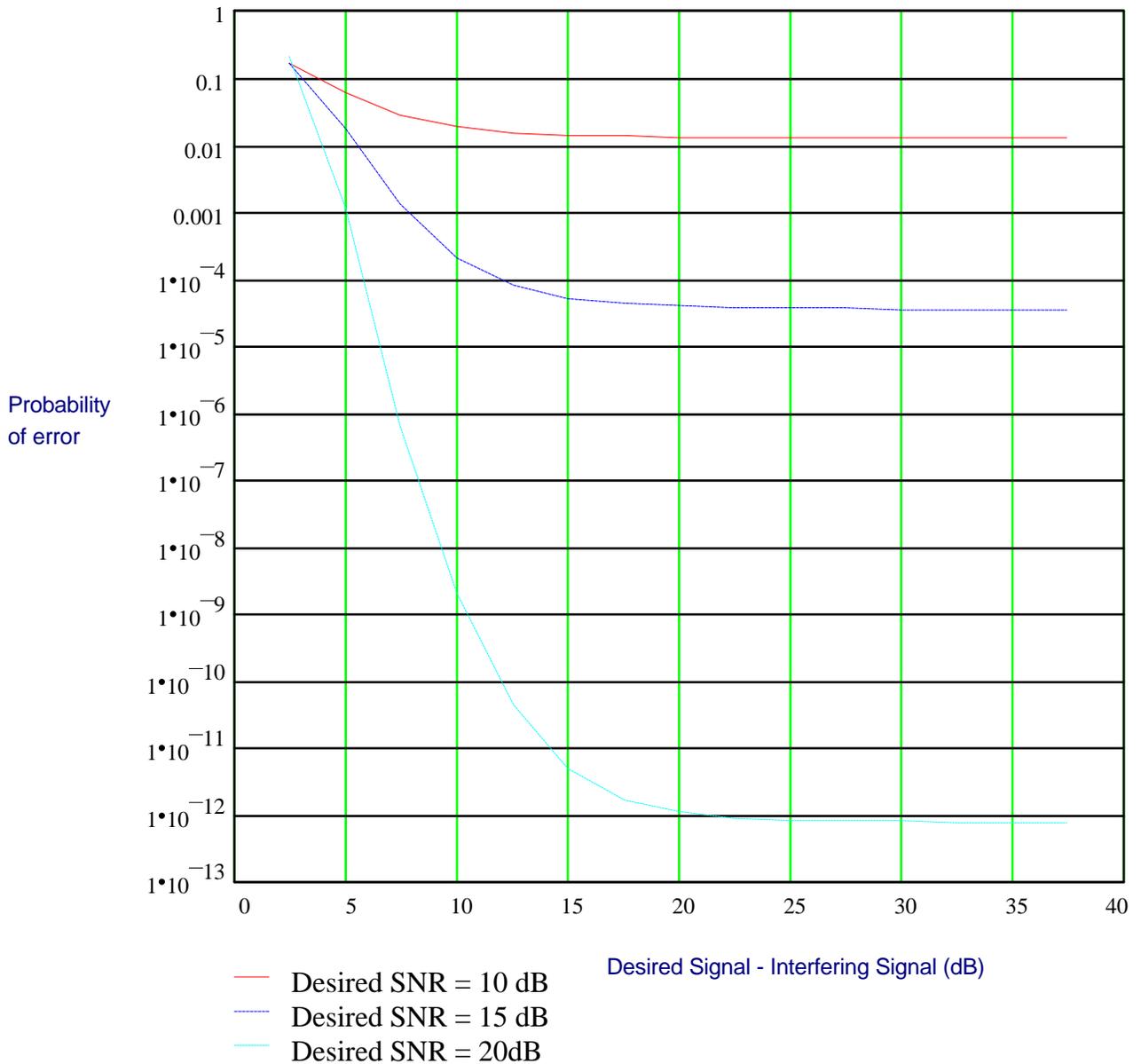and for the case when an interfering signal jams the receiver,

$$P_{jam} := \frac{1}{\sqrt{2 \cdot \pi}} \cdot \int_{\frac{V_t - Si_1}{\sigma}}^{\infty} \exp\left(-\frac{z^2}{2}\right) dz \qquad (8)$$

The total probability of error, using Table 1 and the above expressions, can be stated as follows:

$$Perr := .5 \cdot P_{miss} + .25 \cdot P_{fa} + .25 \cdot P_{jam} \qquad (9)$$

The graph below plots equation 9 for the desired signal having signal to noise ratios of 10, 15, and 20 dB. The x axis indicates the difference between the desired signal level and the interfering signal level in dB (Desired Signal -

Interfering Signal). This graph shows how an interfering signal effects the probability of error for an OOK transmission. As the interfering signal level approaches 20 dB below the desired signal level little noticeable effect is seen on the overall probability of error. The probability of error then assumes the value as if no interfering signal is present.

Plot 2

PROBABILITY OF ERROR FOR ASK

A similar analysis can be done for the case of ASK modulation. The figure below illustrates the ASK modulation in the presence of an interfering signal.
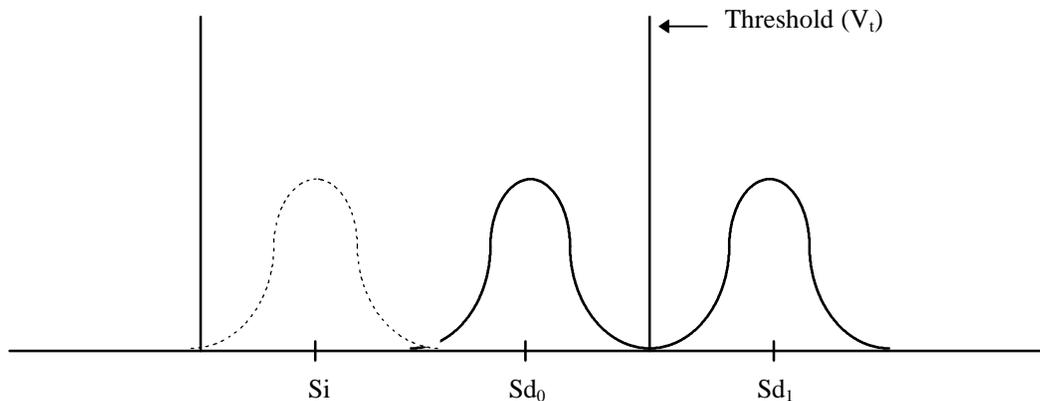

Figure 6

The primary difference between OOK modulation, previously explored, and ASK modulation is that ASK modulation allows for the carrier to be "on" for both the transmission of a "0" and a "1". The carrier, during the transmission of a "0", is reduced in amplitude but is not totally absent as in OOK modulation. The amount of carrier reduction is expressed as a relative level referenced to the maximum level when a "1" is transmitted, usually in dB units. Much of the previous development for evaluating the probability of error for OOK modulation applies directly to ASK modulation. The graph below shows the probability of error for an ASK modulated transmission without an interferer for carrier ("1" to "0") differences of 4.77, 8.4 and 16 dB. This plot indicates that at a given signal to noise ratio, the probability of error improves as the carrier amplitude delta increases between the "1" and "0" states. This is expected, because the greater the difference between the two levels ("0" and "1") the easier it is for the receiver to distinguish between the two levels.
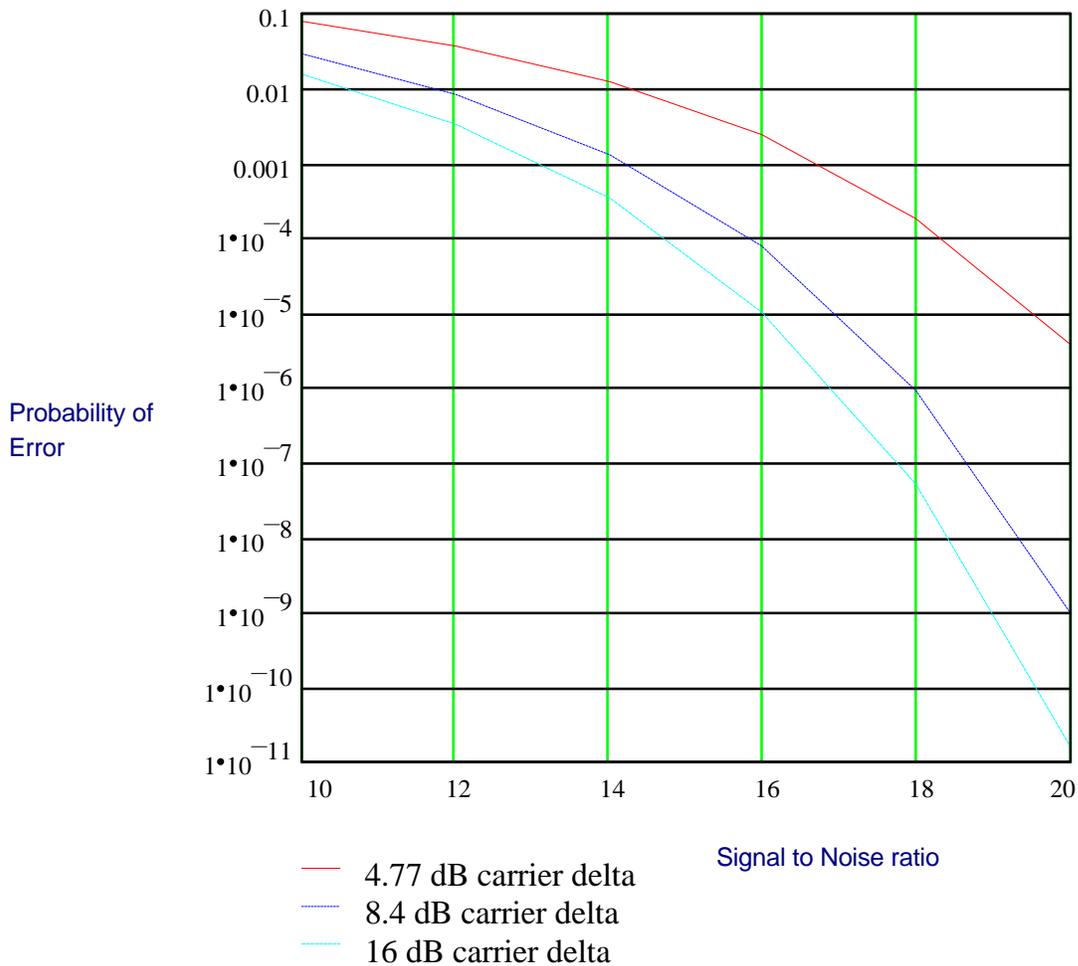
Plot 3

PROBABILITY OF ERROR WITH AN INTERFERING SIGNAL (ASK)

Table 2, below, summarizes the possible error conditions that exist for an ASK modulated signal with an interfering signal present. It is important to remember that the detector is an envelope detector. Therefore, the decision circuitry sees the largest event at any given time, whether the signal is the interferer or the desired signal. As previously stated, the interfering signal and the desired signal are independent and the probability of either transmitting a "1" or "0" is equal. It is also assumed that the interfering signal is OOK modulated.

Transmitted Probability		Receiver Error
$Sd=0; Si=1$	.25	$Si > Sd_0$; & $Si>Vt$; Receiver detects a "1" (Jammed)
		or $Sd_0 > Si$; & $Sd_0>Vt$; Receiver detects a "1" (False alarm)
$Sd=0; Si=0$	.25	$Sd_0 > Vt$; Receiver detects a "1" (False alarm)
$Sd =1; Si=0$	.25	$Sd_1 < Vt$; Receiver detects a "0" (Miss)
$Sd =1; Si=1$	.25	$Sd_1 > Si$; & $Sd_1< Vt$; Receiver detects a "0" (Miss)
		or $Si > Sd_1$; & $Si< Vt$; Receiver detects a "0" (Jammed)

Table 2

$$P(\text{case1}) := .25 \cdot \left[\left[P_{(Si>V_t)} \right]_{givenSi>Sd_0} + \left[P_{(Sd_0>V_t)} \right]_{givenSd_0>Si} \right]$$

$$P(\text{case2}) := .25 \cdot P\left(Sd_0>V_t\right)$$

$$P(\text{case3}) := .25 \cdot P\left(Sd_1<V_t\right)$$

$$P(\text{case4}) := .25 \cdot \left[\left[P_{(Sd_1<V_t)} \right]_{givenSd_1>Si} \right] \qquad (10)$$
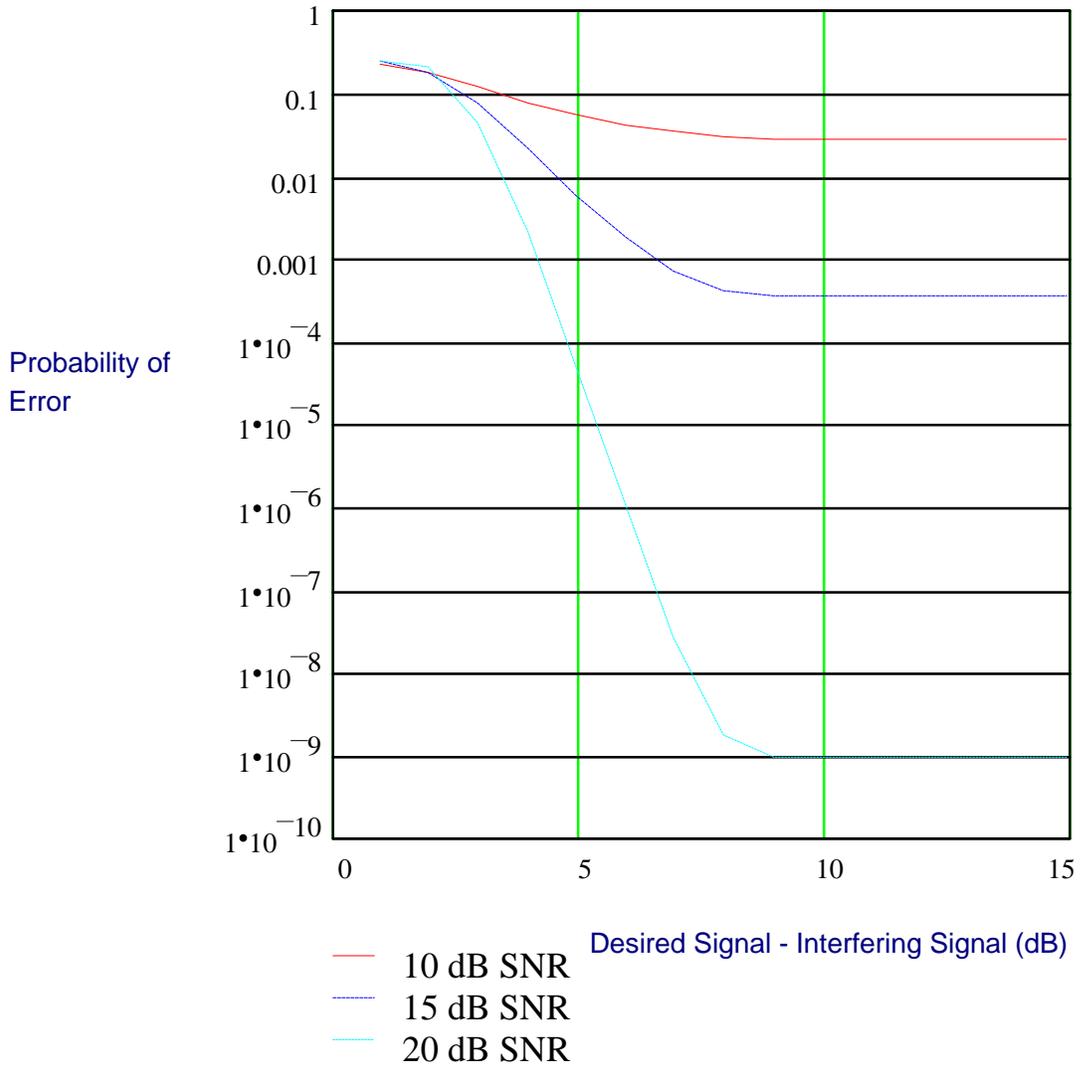
The total probability of error is the sum of the four individual contributing errors identified above in equations 10. The following three plots show the predicted probability of error of an ASK signal with an interfering signal present. Plot 4 is an ASK signal with a delta of 4.77 dB and signal to noise ratios of 10, 15 and 20 dB. Plot 5 is an ASK signal with a delta of 8.4 dB and signal to noise ratios of 10, 15, and 20 dB. Likewise, plot 6 is an ASK signal with a delta of 16 dB. Each of these plots show the effects of the probability of error as a function of the interfering signal level. Comparisons between plots 2 and plots 4, 5, and 6 show that the effects of the interfering signal on the probability of error can be diminished by transmitting ASK versus OOK. Plots 4, 5, and 6 also show that decreasing the delta between the two carrier levels for a "1" or a "0" diminish the effect caused by the interfering signal. However, The trade-off, displayed in plot 3, is that the probability of error worsens as the delta is decreased.

4.77 dB delta ASK signal
with an interfering signal

Probability of
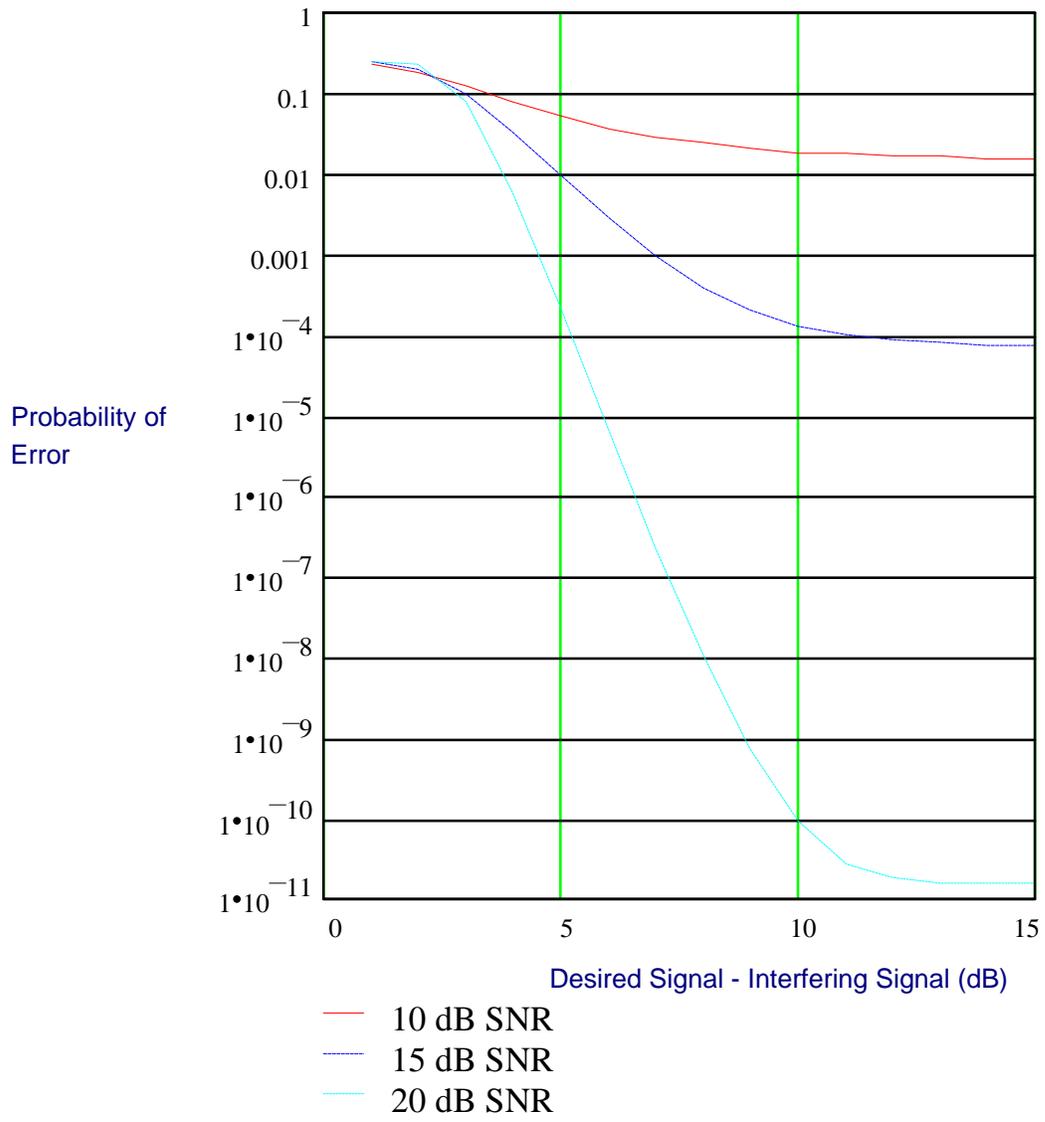Error

Desired Signal - Interfering Signal (dB)

10 dB SNR
15 dB SNR
20 dB SNR
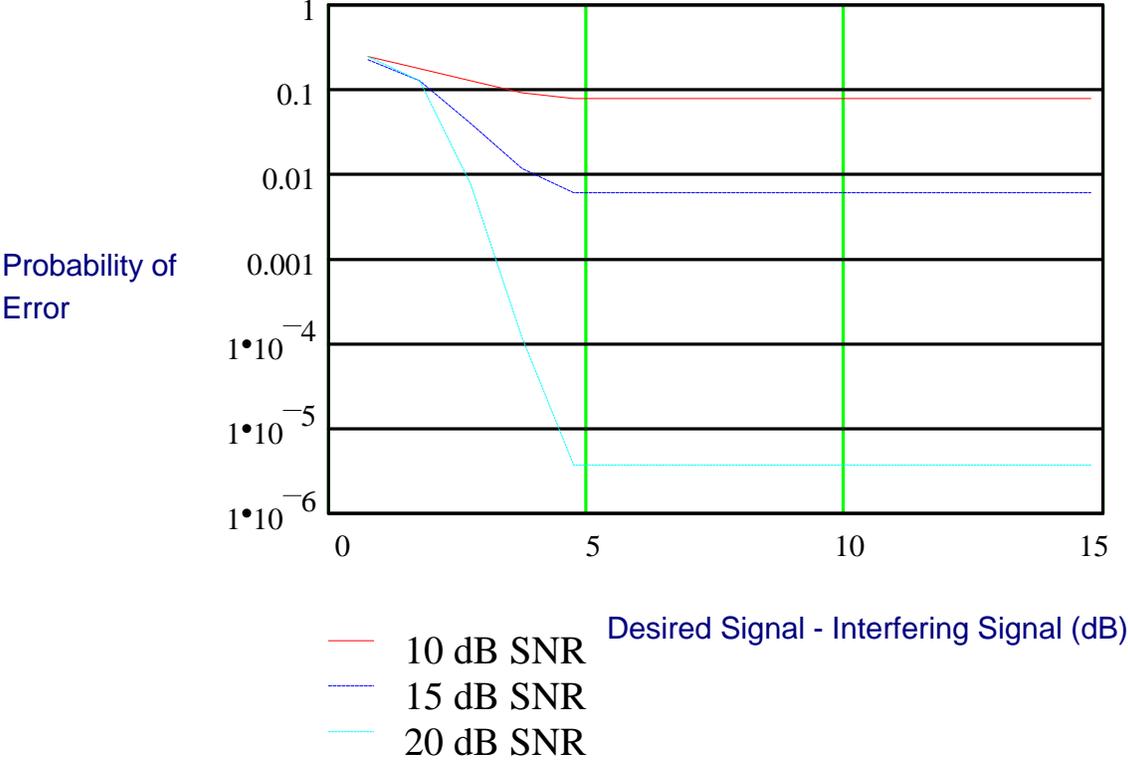
Plot 4

8.4 dB delta ASK signal
with an interfering signal

Probability of Error

Desired Signal - Interfering Signal (dB)

10 dB SNR
15 dB SNR
20 dB SNR

Plot 5

16 dB delta   ASK signal
with an interfering signal

Probability of Error — vertical axis:
1
0.1
0.01
0.001
$1 \cdot 10^{-4}$
$1 \cdot 10^{-5}$
$1 \cdot 10^{-6}$
$1 \cdot 10^{-7}$
$1 \cdot 10^{-8}$
$1 \cdot 10^{-9}$
$1 \cdot 10^{-10}$
$1 \cdot 10^{-11}$

Desired Signal - Interfering Signal (dB)

—— 10 dB SNR
---- 15 dB SNR
—— 20 dB SNR

Plot 6

PROBABILITY OF ERROR FOR FSK

The probability of error for a non-coherent FSK modulation was previously described by formula 3 and graphed on plot 1. The model used is shown below in figure 7.
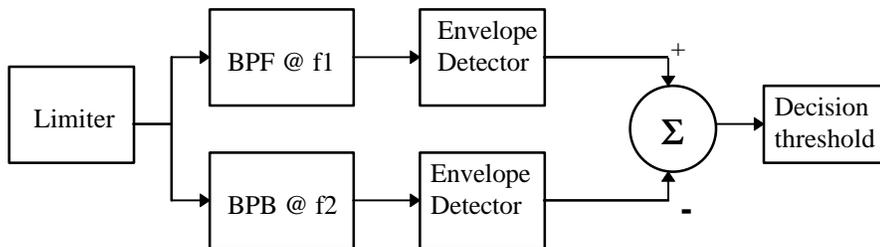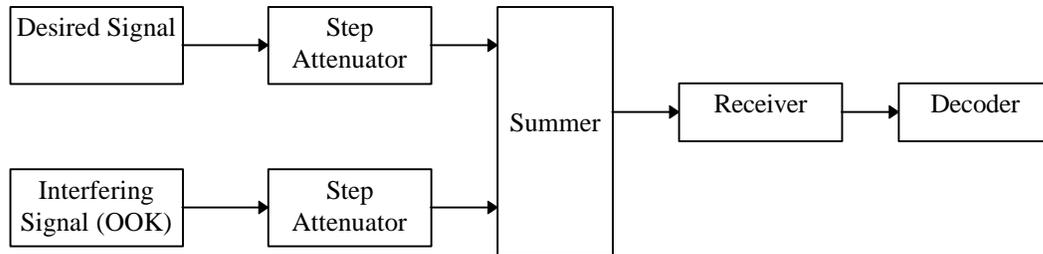

This model suggests that non-coherent FSK modulation can be treated as two ASK signals, one at frequency f1 and the other at frequency f2. Since the limiter theoretically keeps the amplitude of the signals at f1 and f2 constant, the probability distribution appears as follows:
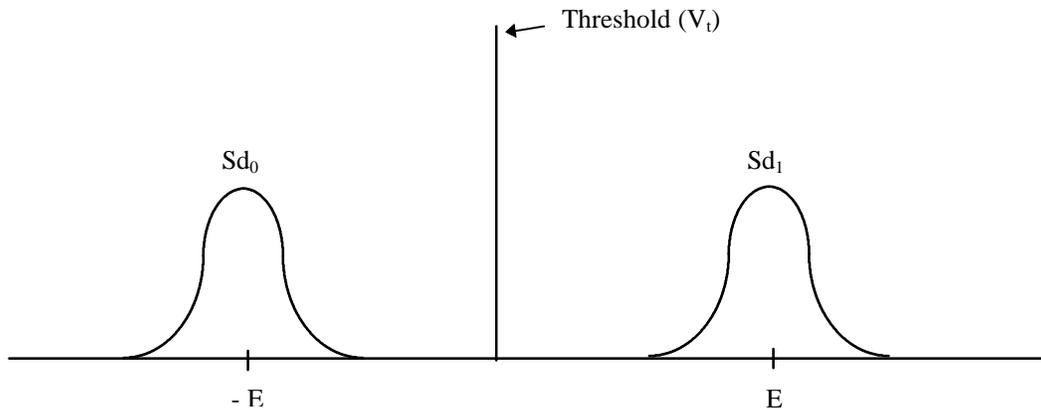
Using the assumption, previously used, that the noise of the interferer and the noise of the desired signal are perfectly correlated, the interfering signal should not influence the probability of error until it is greater than the desired signal. This assumption is valid since the two signals are present in the same narrow band filter of the receiver at the same time.   Since the signal, for FSK modulation, is present at the same level for a "0" and a "1" transmission, it is intuitive that FSK modulation could provide marginal improvement over ASK modulation in the presence of an interferer.

LABORATORY RESULTS

The tests were conducted using a standard encoder/decoder pair that required 36 bits of error free data in order to indicate a valid response. The data was transmitted at 2.4 Kbps. The test set up is shown below:

```
┌────────────────┐      ┌────────────┐
│ Desired Signal │ ───▶ │    Step    │ ───▶
└────────────────┘      │ Attenuator │       ┌──────────┐
                        └────────────┘       │          │ ──▶ ┌──────────┐ ──▶ ┌─────────┐
                                             │  Summer  │      │ Receiver │      │ Decoder │
┌────────────────┐      ┌────────────┐       │          │      └──────────┘      └─────────┘
│   Interfering  │ ───▶ │    Step    │ ───▶  └──────────┘
│  Signal (OOK)  │      │ Attenuator │
└────────────────┘      └────────────┘
```

The tables below indicates the level of the desired signal and the level at which the interferer degraded the receivers' performance by approximately 50 percent.

Desired Signal Modulation - OOK   Minimum Sensitivity -92 dBm

Desired Signal Level (OOK)	Interferer Signal Level (OOK)	Delta (Desired - Interferer) dB
-20	-70	50
-25	-70	45
-30	-72	42
-35	-75	40
-40	-75	35
-45	-75	30
-50	-77	27
-55	-78	23
-60	-79	19
-65	-81	16
-70	-82	12
-75	-83	8
-80	-86	6
-85	-90	5
-90	-96	6

Desired Signal Modulation - ASK Delta 4.77dB   Minimum Sensitivity -90 dBm

Desired Signal Level (ASK)	Interferer Signal Level (OOK)	Delta (Desired - Interferer) dB
-20	-21	1
-25	-26	1
-30	-35	5
-35	-38	3
-40	-43	3
-45	-55	10
-50	-53	3
-55	-58	3
-60	-70	10
-65	-69	4
-70	-75	5
-75	-84	9
-80	-95	15
-85	-101	16
-90	-101	11

Desired Signal Modulation - ASK Delta 9dB   Minimum Sensitivity -93 dBm

Desired Signal Level (ASK)	Interferer Signal Level (OOK)	Delta (Desired - Interferer) dB
-20	-24	4
-25	-31	6
-30	-35	5
-35	-40	5
-40	-46	6
-45	-50	5
-50	-55	5
-55	-61	6
-60	-65	5
-65	-70	5
-70	-76	6
-75	-82	7
-80	-86	6
-85	-91	6
-90	-101	11

Desired Signal Modulation - ASK Delta 16dB   Minimum Sensitivity -95 dBm

Desired Signal Level (ASK)	Interferer Signal Level (OOK)	Delta (Desired - Interferer) dB
-20	-31	11
-25	-36	11
-30	-41	11
-35	-47	12
-40	-51	11
-45	-56	11
-50	-62	12
-55	-66	11
-60	-70	10
-65	-77	12
-70	-81	11
-75	-84	9
-80	-86	6
-85	-90	5
-90	-95	5

Desired Signal Modulation - FSK Delta 62KHz   Minimum Sensitivity -92 dBm

Desired Signal Level (FSK)	Interferer Signal Level (OOK)	Delta (Desired - Interferer) dB
-20	-26	6
-25	-32	7
-30	-36	6
-35	-40	5
-40	-45	5
-45	-50	5
-50	-55	5
-55	-58	3
-60	-63	3
-65	-68	3
-70	-74	4
-75	-80	5
-80	-86	6
-85	-91	6
-90	-98	8

The above data clearly shows less interference influence with the ASK modulation compared with that of the OOK modulation. Also, as predicted, the results show better sensitivity and greater influence of the interferer as the ASK delta is increased.   The lab results indicate that the ASK modulation is as good or better than the FSK modulation in performance with an interfering signal. This is contrary to intuition and theory. The trends of the actual data seem to match closely with what was predicted. However, due to the fact that the actual receiver components do not behave perfectly, anomalies can be seen in the data. Some examples of non-ideal behavior could be log-amp non-linearities, receiver saturation, and noise not being perfectly correlated as theorized. The model also does not take into account that if the interfering signal is to close in frequency with the desired signal a beat note will be present within the detection pass band of the receiver. This will cause the interfering signal to have an even stronger influence on the receiver.

CONCLUSION

ASK modulation is an alternative to OOK and FSK modulation. ASK modulation offers the advantage of being more immune to interfering signals than OOK and is easier to implement at a lower cost than FSK modulation. RFM will be offering a low cost transceiver chip that will allow for both OOK and ASK modulation in the late 1998 time frame.

ACKNOWLEDGMENTS

[1] Peebles, Jr., Peyton Z., Digital Communication Systems, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1987.